Math 157 - Test 3a
November 19, 2014

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary.

1. (14 points) Let $f(x) = 2x^3 - 3x^2 - 36x + 10$. Find open intervals on which f is increasing/decreasing. Then find and classify all local (relative) extreme values.

$$f'(x) = 6x^2 - 6x - 36$$
$$= 6(x^2 - x - 6)$$
$$= 6(x-3)(x+2) = 0$$

$\Rightarrow x = 3$ OR $x = -2$

SIGNS OF $f'(x)$: $+$ on $(-\infty, -2)$, $-$ on $(-2, 3)$, $+$ on $(3, \infty)$

INC DEC INC

f IS INCREASING ON $(-\infty, -2) \cup (3, \infty)$

f IS DECREASING ON $(-2, 3)$

$f(-2) = 54$ IS A REL MAX

$f(3) = -71$ IS A REL MIN

2. (8 points) In solving an optimization problem, Joe found that $x = 1$ is a critical number of the function $P(x) = 2x + \frac{2}{x}$. Use calculus to show that Joe's critical number minimizes P.

$$P(x) = 2x + 2x^{-1}$$
$$P'(x) = 2 - 2x^{-2}$$
$$= 2 - \frac{2}{x^2}$$

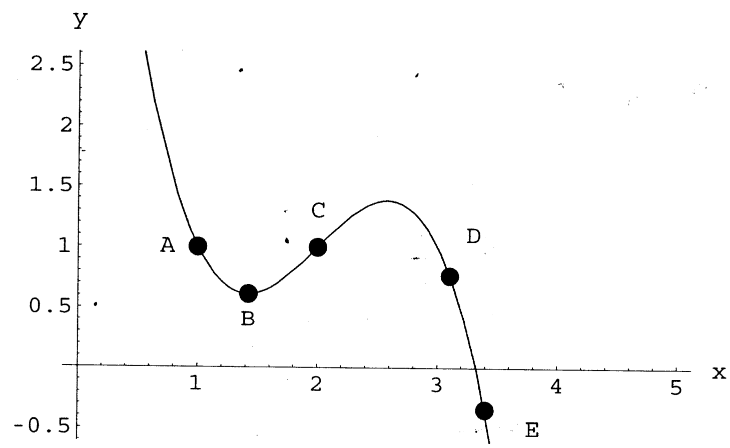
$P'(x) = 0 \Rightarrow x = \pm 1$

$P'(x)$ DNE WHEN $x = 0$

SIGNS OF $P'(x)$: $+$ on $(-\infty, -1)$, $-$ on $(-1, 0)$, $-$ on $(0, +1)$, $+$ on $(+1, \infty)$

$x = 1$ GIVES A RELATIVE MIN.

3. (6 points) The graph of f is shown below. For each part of this problem, find a labeled point that satisfies the given condition.



(a) $f''(x) = 0$ C

(b) $f'(x) = 0$ B

(c) $f''(x) < 0$ D, E

(d) $f(x) < 0$ E

(e) $f'(x) > 0$ C

(f) $f''(x) > 0$ A, B

4. (6 points) When the production level is 5000 units, marginal revenue is \$5.15 per unit and marginal cost is \$5.75 per unit. Do you expect maximum profit to occur at a production level above or below 5000 units? Explain your reasoning.

$P(q) := R(q) - C(q)$

WHEN $q = 5000$,

$P'(5000) = R'(5000) - C'(5000)$

$= 5.15 - 5.75 = -0.60 < 0$

$\Rightarrow$ PROFIT IS DECREASING WHEN $q = 5000$

$\Rightarrow$ MAX PROFIT BELOW $q = 5000$

5. (8 points) Find the inflection point(s) of the graph of $g(x) = xe^{-x}$.

$$g'(x) = e^{-x} - xe^{-x}$$

$$g''(x) = -e^{-x} - e^{-x} + xe^{-x}$$

$$= e^{-x}(-2 + x)$$

$$g''(x) = 0 \Rightarrow x = 2$$

SIGNS OF $g''(x)$

$-$ $\quad$ $+$

$\longleftarrow$ 2 $\longrightarrow$

$\underbrace{\text{CD} \quad 2 \quad \text{CU}}_{\text{CONCAVITY CHANGES!}}$

$\left(2, \frac{2}{e^2}\right)$ IS THE ONLY INFLECTION PT.

6. (8 points) Find the global (absolute) extreme values of $f(x) = 2x^3 - 9x^2 + 12x$ on the interval $-0.5 \leq x \leq 3$.

$$f'(x) = 6x^2 - 18x + 12$$

$$= 6(x - 3x + 2)$$

$$= 6(x-2)(x-1) = 0$$

$$x = 2, \ x = 1$$

CHECK CRIT #'S AND ENDPTS

$f(2) = 4$

$f(1) = 5$

$\boxed{\begin{array}{l} f(-0.5) = -8.5 \leftarrow \text{ABS MIN} \\ f(3) = 9 \leftarrow \text{ABS MAX} \end{array}}$

7. (6 points) Suppose f has a continuous derivative whose values are given in the table below.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f'(x)$	6	2	1	-1	-4	-5	-2	1	3	2	-1

f INC ★ f DEC ★ f INC ★ f DEC

(a) Find reasonable estimates for the critical numbers of f.

$f'(x) = 0$ AT ABOUT $\boxed{x = -2.5,\ x = 1.5,\ x = 4.5}$

(b) Determine whether each one of your critical numbers gives a local (relative) minimum or maximum. Briefly explain how you know.

f IS INCREASING WHERE $f'(x)$ IS POSITIVE

f IS DECREASING WHERE $f'(x)$ IS NEGATIVE

$\Rightarrow$ $x = -2.5$ GIVES A MAX, $x = 1.5$ GIVES A MIN, $x = 4.5$ GIVES A MAX

8. (8 points) The revenue from selling q items is $R(q) = 800q - q^2$, and the total cost is $C(q) = 150 + 12q$. Find the quantity that maximizes profit.

$$P(q) = R(q) - C(q) = (800q - q^2) - (150 + 12q)$$

$$= 788q - q^2 - 150$$

$$P'(q) = 788 - 2q = 0$$

$$\Rightarrow q = 394$$

SINCE $P''(q) = -2 < 0$,

THE GRAPH OF P IS ALWAYS CONCAVE DOWN $\Rightarrow$ $q = 394$ GIVES THE GLOBAL MAX

9. (8 points) The velocity, v, of an object at time t is described in the table below.

t (sec)	0	1	2	3	4	5	6
v (ft/sec)	3	6	10	16	22	20	18

(a) Use a right sum with $\Delta t = 2$ to estimate the total distance traveled by the object.

DISTANCE $\approx 10(2) + 22(2) + 18(2)$

$= \boxed{100 \text{ FT}}$

(b) Use a left sum with $\Delta t = 1$ to estimate the total distance traveled by the object.

DISTANCE $\approx 3(1) + 6(1) + 10(1) + 16(1) + 22(1) + 20(1)$

$= \boxed{77 \text{ FT}}$

(c) Which of your approximations do you think better estimates the distance traveled? Why?

(b) USES MORE DATA AND SMALLER TIME SUBINTERVALS. IT IS PROBABLY BETTER.

10. (8 points) Use a left sum with 4 subintervals (rectangles) of equal width to estimate $\int_0^1 e^{-x^2}\,dx$.

$\Delta X = \frac{1-0}{4} = \frac{1}{4} = 0.25$

X	0	0.25	0.5	0.75	1
e^{-x^2}	1	0.9394	0.7788	0.5698	—

LEFT SUM $= 1(0.25) + 0.9394(0.25)$
$+ 0.7788(0.25) + 0.5698(0.25)$
$= \boxed{0.822}$

11. (5 points extra credit) The quantity of a drug in the bloodstream t hours after a tablet is swallowed is given, in milligrams, by

$$Q(t) = 25(e^{-t} - e^{-2t}).$$

What is the maximum quantity of the drug in the bloodstream?

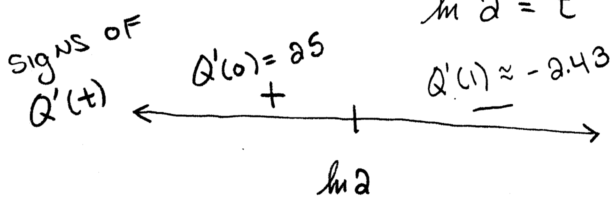
$$Q'(t) = 25\left(-e^{-t} + 2e^{-2t}\right) = 0$$

$$\Rightarrow 2e^{-2t} = e^{-t}$$

$$\Rightarrow 2 = e^{t}$$

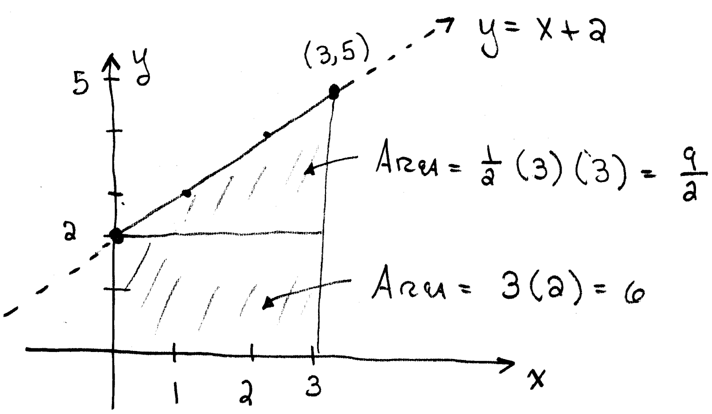
$$\ln 2 = t$$

$Q(\ln 2) = 6.25$ mg IS THE MAX.



$t = \ln 2$ GIVES A MAX

12. (5 points extra credit) Sketch the graph of $f(x) = x + 2$ over the interval from $x = 0$ to $x = 3$. Then use area to compute the exact value of $\int_0^3 (x+2)\,dx$. Show all work or explain your reasoning.



$$\int_0^3 (x+2)\,dx = \frac{9}{2} + 6 = \boxed{10.5}$$

Math 157 - Test 3b
November 19, 2014

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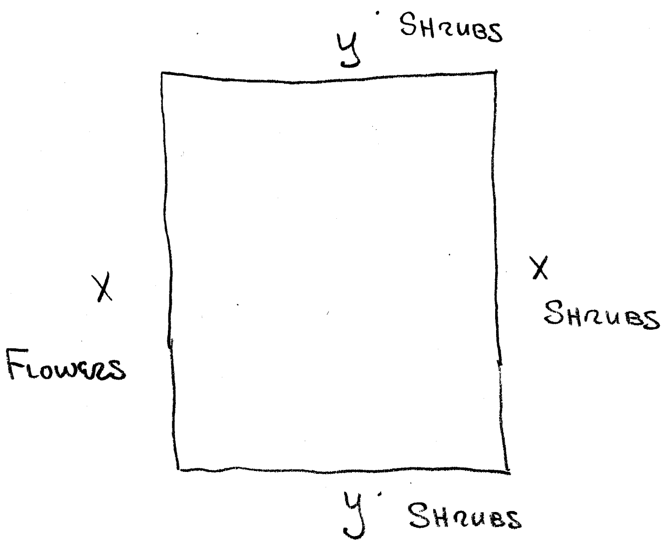
Show all work to receive full credit. Supply explanations where necessary. YOU MUST WORK INDIVIDUALLY ON THIS EXAM.

1. (10 points) A landscape architect plans to enclose a 4000 square-foot rectangular region in a botanical garden. She will use shrubs costing \$30 per foot along three sides and flowers costing \$10 per foot along the fourth side. Determine a function giving the total cost of the project and then find the minimum cost.



$xy = 4000 \text{ FT}^2$

$\text{Cost} = \underbrace{30y + 30x + 30y + 10x}_{40x + 60y}$

Minimize $\text{Cost} = 40x + 60y$

s.t. $xy = 4000$

$y = \frac{4000}{x}$

$$\boxed{\text{Cost} = C(x) = 40x + \frac{240000}{x}}$$

$C'(x) = 40 - \frac{240000}{x^2} = 0$

$\Downarrow$

$\frac{240000}{x^2} = 40$

$\Downarrow$

$x^2 = 6000$

$\Downarrow$

$x = \sqrt{6000}$

$C''(x) = \frac{480000}{x^3}$

$C''(\sqrt{6000}) > 0$

$\Downarrow$

$x = \sqrt{6000}$ gives a min.

$x = \sqrt{6000} \approx 77.46$ FT

$y = \frac{4000}{\sqrt{6000}} \approx 51.64$ FT

$\text{Cost} = 40(\sqrt{6000}) + \frac{240000}{\sqrt{6000}}$

$\approx \boxed{\$6196.77}$

2. (7 points) Use a left or right sum with 8 subintervals (rectangles) of equal width to estimate $\int_0^2 \frac{1}{1+t^2}\, dt$. Then use your calculator to estimate the value of the definite integral.

$\Delta t = \frac{2-0}{8} = 0.25$. $f(t) = \frac{1}{1+t^2}$

PARTITION : $0 < 0.25 < 0.5 < 0.75 < 1 < 1.25 < 1.5 < 1.75 < 2$

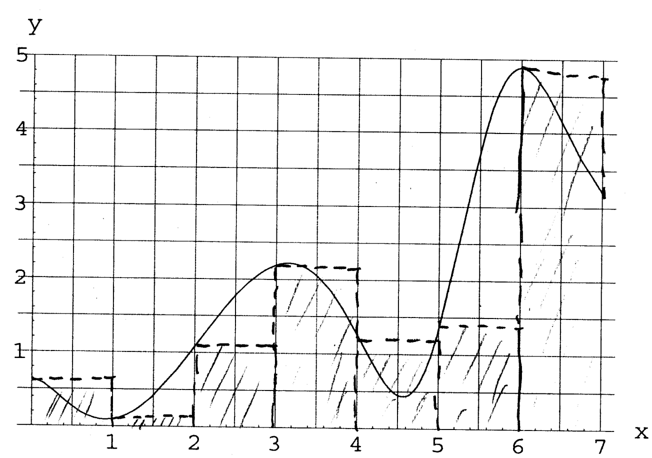
LEFT SUM $= 0.25\left[f(0) + f(0.25) + \dots + f(1.75)\right]$

≈ 1.2063

RIGHT SUM $= 0.25\left[f(0.25) + f(0.5) + \dots + f(2)\right]$

≈ 1.1076

3. (3 points) The graph of f is shown below. Find a reasonable estimate for $\int_0^7 f(x)\, dx$.



I'LL USE A LEFT SUM WITH $\Delta X = 1$ ...

$(0.6)(1) + (0.1)(1) + (1.1)(1) + (2.2)(1)$
$+ (1.2)(1) + (1.4)(1) + (4.9)(1)$
$= \boxed{11.5}$