## Math 157 - Test 2

October 15, 2014

Name _____

Score _____

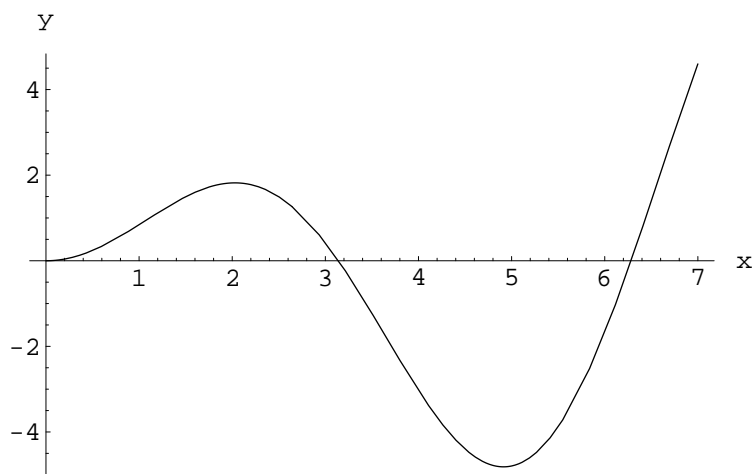**Show all work to receive full credit.** Supply explanations where necessary.

1. (8 points) Find an equation of the line tangent to the graph of

$$f(x) = x^3 - 5x^2 + 4x + 6$$

at the point where $x = 1$.

2. (5 points) The graph of the function $f$ is shown below. Referring to this graph, arrange the following quantities in ascending order.

$$f'(1), \quad f'(5), \quad f(5), \quad f(7), \quad f'(3)$$

3. (20 points) Determine the derivative of each function.

(a) $y = 8t^3 - 4t^2 + 12t - 3$

(b) $f(x) = \sqrt{\dfrac{1}{x^3}}$

(c) $P = \ln(t^2 + 1)$

(d) $w = (5r^2 - 8)^4$

(e) $g(x) = (x^3 - 7x^2 + 1)e^x$

4. (7 points) A glass of water is placed in a hot, dry room where the water begins to quickly evaporate. The height, in centimeters, of the water in the glass after $t$ hours is given by
$$h(t) = 20 - 0.094t^2.$$
Compute $h(6)$ and $h'(6)$. Using units, explain what each of these values represents.

5. (6 points) Given that $f(1) = 4$ and $f'(1) = 5$, find $g'(1)$ if $g(x) = \sqrt{f(x)}$.

6. (7 points) Find the critical numbers of $f(x) = (x^2 - 4)^3$.

7. (7 points) Since the beginning of 1990, the population of Aurora, IL has been approximately given by
$$P(t) = 101(1.03)^t,$$
where $t$ represents years since 1990 and $P$ is in thousands of people. At what rate will the population be changing in 2025? Include units with your answer.
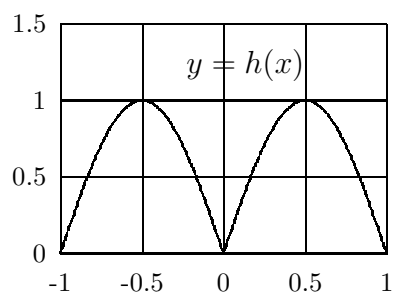
8. (12 points) Find the second derivative of each function.

(a) $h(x) = x^4 + 5x^2 + e^{-3x}$

(b) $r(x) = 2x + 2\ln x$

9. (7 points) Find the instantaneous rate of change of $y = x^2 e^{5x}$ at the point where $x = -1$.

10. (7 points) Consider the function $y = h(x)$ whose graph is shown below. The domain of $h$ is the closed interval $[-1, 1]$. Find the critical numbers of $h$ and explain why they are critical numbers.

11. (8 points) The table below gives the values of the functions $f$ and $g$ and their derivatives at selected values of $x$.

| $x$ | $-2$ | $-1$ | $2$ |
|---|---|---|---|
| $f(x)$ | $1$ | $3$ | $-2$ |
| $f'(x)$ | $2$ | $-1$ | $-1$ |
| $g(x)$ | $2$ | $0$ | $-2$ |
| $g'(x)$ | $-3$ | $-2$ | $1$ |

(a) If $h(x) = f(x) \cdot g(x)$, use the product rule to compute $h'(-1)$.

(b) If $h(x) = \dfrac{f(x)}{g(x)}$, use the quotient rule to compute $h'(2)$.

12. (6 points) For a function $f(x)$, we know that $f(10) = 8$ and $f'(10) = 3$. Use this information to find a reasonable estimate for $f(9)$.

13. (5 points extra credit) Find the critical numbers of $g(x) = (x-2)^2(2x-9)$.

14. (5 points extra credit) Find the second derivative of $w = \dfrac{3y - y^2}{5 + y}$.