Math 157 - Test 1
September 17, 2014

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary.

1. (9 points) The table below shows U.S. alternative and nuclear energy usage as a percent of total energy usage.

Year	1960	1970	1980	1990	2000	2010
Percent of Total Energy Usage	1.26	1.81	5.43	10.30	10.77	11.72

(a) Compute the average rate of change from 1960 to 2010.

$$\frac{11.72 - 1.26}{2010 - 1960} = \frac{10.46}{50} = \boxed{0.2092 \approx 0.21\% \text{ PER YEAR}}$$

(b) Estimate the instantaneous rate of change in the 1990.

LOOK AT SMALLEST INTERVAL AROUND 1990.

LOOK AT [1980, 2000]...

$$\frac{10.77 - 5.43}{20} = \frac{5.34}{20} = \boxed{0.267 \approx 0.27\% \text{ per year}}$$

(c) Find the relative change in the percent of total energy usage from 1980 to 2010.

$$\frac{11.72 - 5.43}{5.43} = \frac{6.29}{5.43} \approx \boxed{1.158 \approx 116\% \text{ INCREASE}}$$

2. (4 points) Find an equation of the line passing through $(2, 3)$ and $(-4, 6)$.

$$m = \frac{6-3}{-4-2} = \frac{3}{-6} = -\frac{1}{2}$$

$$y = -\frac{1}{2}x + b$$

$$3 = -\frac{1}{2}(2) + b$$

$$3 = -1 + b \Rightarrow b = 4$$

$$\boxed{y = -\frac{1}{2}x + 4}$$

3. (6 points) Use a table of values to estimate the following limit. Your table must show function values at six or more points.

x	$\frac{x^2-x-12}{2-\sqrt{x}}$
4.1	-28.5764
3.9	-27.4264
4.01	-28.0575
3.99	-27.9425
4.001	-28.0058
3.999	-27.9943

$$\lim_{x\to 4}\frac{x^2-x-12}{2-\sqrt{x}} = \boxed{-28}$$

IT LOOKS LIKE

4. (4 points) Let $f(x) = 2x^3 - x$. Compute the average rate of change of f from $x = 1$ to $x = 3$.

$$\frac{\Delta y}{\Delta x} = \frac{f(3)-f(1)}{3-1} = \frac{51-1}{2} = \boxed{25}$$

5. (6 points) Solve for t.

(a) $100 = 25(1.5)^t$

$$\frac{100}{25} = \frac{25(1.5)^t}{25} \Rightarrow 4 = 1.5^t$$

$$\ln 4 = t \ln(1.5)$$

$$t = \frac{\ln 4}{\ln 1.5} \approx 3.419$$

(b) $5e^{3t} = 8e^{2t}$

$$e^{3t} = \frac{8}{5}e^{2t}$$

$$e^t = \frac{8}{5}$$

$$t = \ln\frac{8}{5} \approx 0.47$$

Exp Decay: $P(t) = P_0 e^{kt}$

6. (6 points) If the quantity of a substance decreases by 4% in 10 hours, find its half-life.

96% REMAINS AFTER 10 HOURS

$\Rightarrow \quad 0.96 P_0 = P_0 e^{k(10)}$

$0.96 = e^{10k}$

$\ln 0.96 = 10k$

$\dfrac{\ln 0.96}{10} = k$

HALF-LIFE $= t = \dfrac{\ln \frac{1}{2}}{k}$

$t = \dfrac{\ln \frac{1}{2}}{\frac{\ln 0.96}{10}} \approx \boxed{170 \text{ HRS}}$

7. (6 points) Use algebra to find the limit.

$(3+x)^2 = 9 + 6x + x^2$

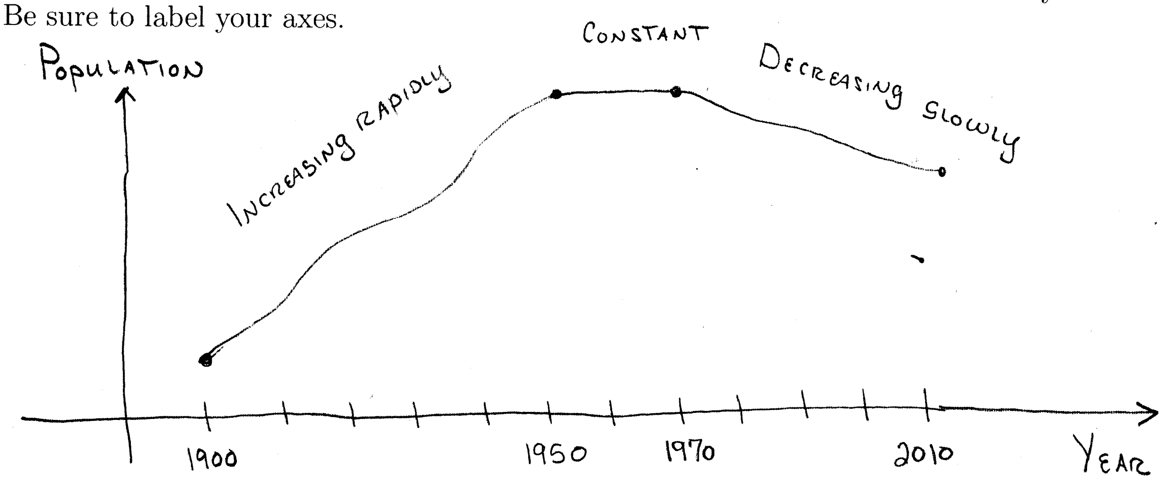
$$\lim_{x\to 0} \frac{(3+x)^2 - 9}{x}$$

$$\lim_{x\to 0} \frac{(9+6x+x^2)-9}{x} = \lim_{x\to 0} \frac{6x+x^2}{x}$$

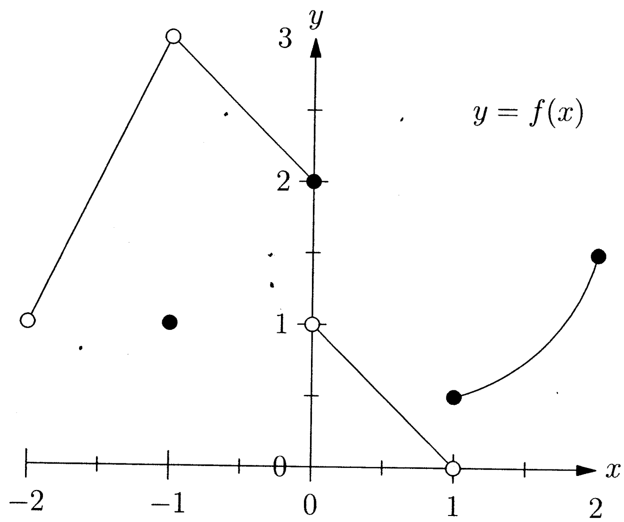
$$= \lim_{x\to 0} \frac{x(6+x)}{x} = \lim_{x\to 0} (6+x)$$

$$= \boxed{6}$$

8. (5 points) The population of Silver Oaks grew rapidly from 1900 to 1950, stayed roughly constant from 1950 to 1970, and then decreased slowly until 2010. Sketch the graph of a function that could describe the population of Silver Oaks as a function of the year. Be sure to label your axes.



9. (12 points) The graph of the function f is shown below.



(a) On the interval $-2 < x < 2$, at which points is f discontinuous?

DISCONTINUOUS AT $X=-1$, $X=0$, AND $X=1$

(b) Briefly explain why $\lim_{x\to 0} f(x)$ fails to exist.

FROM DIFFERENT SIDES OF $X=0$, THE FUNCTION VALUES GET CLOSER TO DIFFERENT NUMBERS.

(c) Estimate the value of $\lim_{x\to -1} f(x)$.

≈ 3

(d) Estimate the value of $f(-1)$.

≈ 1

(e) Compute the average rate of change over the interval from $x = 1$ to $x = 2$.

$$\frac{f(2)-f(1)}{2-1} \approx \frac{1.5-0.5}{1} = \frac{1}{1} = 1$$

(f) Estimate the instantaneous rate of change of f at $x = 0.5$.

THE SLOPE AT $X=0.5$ IS -1

10. (4 points) Could this data be representative of a linear function? Show work.

x	1	2	5	7	15
y	14	12	6	1	-12

Δx: 1, 3, 2, 8

Δy: -2, -6, -5, -13

$$\frac{-2}{1} = -2$$

$$\frac{-6}{3} = -2$$

$$\frac{-5}{2} \neq -2$$

THIS DATA IS <u>NOT</u> LINEAR.

11. (8 points) Let $g(x) = 5^x$. Use at least four small intervals to estimate $g'(1)$.

$[1, 1.1] \rightarrow \dfrac{5^{1.1} - 5^1}{0.1} \approx 8.73$

$[0.9, 1] \rightarrow \dfrac{5^{0.9} - 5^1}{-0.1} \approx 7.43$

$[1, 1.001] \rightarrow \dfrac{5^{1.001} - 5^1}{0.001} \approx 8.05$

$[0.999, 1] \rightarrow \dfrac{5^{.999} - 5^1}{-0.001} \approx 8.04$

LOOKS LIKE $g'(1) \approx 8.045$

12. (5 points) A city's population was 30,700 in the year 2010 and is growing linearly by 850 people per year. Find a formula for the city's population, P, as a function of the number of years, t, since 2010.

$m = 850$

INITIAL POP = Y-INT = 30,700

$$\boxed{P = 850t + 30700}$$

13. (5 points) Which is a bigger relative change: an increase of class size from 5 to 10 or increase in class size from 30 to 50? Show work to justify your answer.

5 TO 10

$$\frac{10-5}{5} = \frac{5}{5} = 1 = \boxed{100\%}$$

30 TO 50

$$\frac{50-30}{30} = \frac{20}{30} = \frac{2}{3} \approx \boxed{66.7\%}$$

$\boxed{\text{5 TO 10 IS BIGGER}}$

14. (10 points) The population of the U.S. was 281.4 million in 2000 and 308.7 million in 2010. Assume exponential growth.

(a) Find a model of the form $P(t) = P_0 e^{kt}$ that gives the U.S. population as a function of years, t, since 2000.

$$P(t) = 281.4\, e^{kt}$$

$$P(10) = 308.7 = 281.4\, e^{k(10)}$$

$$\Rightarrow \frac{308.7}{281.4} = e^{10k}$$

$$\Rightarrow \ln\left(\frac{308.7}{281.4}\right) = 10k$$

$$\Rightarrow \frac{1}{10} \ln\left(\frac{308.7}{281.4}\right) = k \approx 0.00926$$

$$\boxed{P(t) = 281.4\, e^{0.00926t}}$$

(b) What is the continuous growth rate of the U.S. population?

$$k \approx 0.00926$$

$$\Rightarrow \boxed{0.926\%}$$

(c) In what year will the U.S. population reach 400 million?

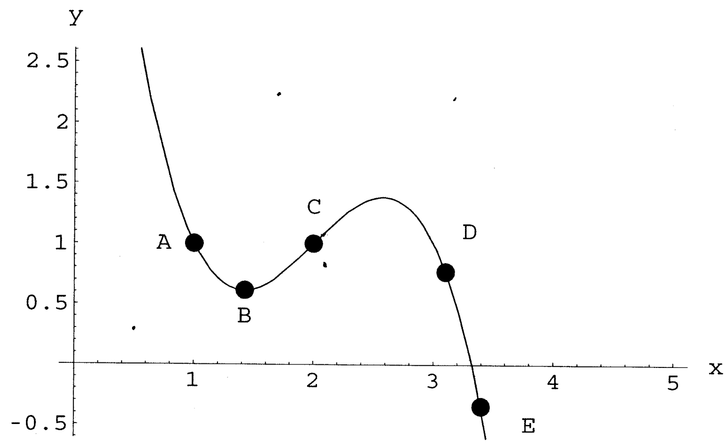
$$400 = 281.4\, e^{0.00926t}$$

$$\frac{400}{281.4} = e^{0.00926t}$$

$$\Rightarrow t = \frac{\ln\left(\frac{400}{281.4}\right)}{0.00926}$$

$$\approx 37.979 \Rightarrow \boxed{2038}$$

15. (10 points) The graph of the function f is shown below.



(a) At which of the labeled points is it true that $\underbrace{f'(x)}_{\text{NEGATIVE SLOPE}} < 0$?

A, D, E

(b) At which of the labeled points is it true that $\underbrace{f'(x) > 0}_{\text{POSITIVE SLOPE}}$?

C

(c) At which of the labeled points is it true that $\underbrace{f'(x) = 0}_{\text{HORIZONTAL TAN LINE}}$?

B

(d) At which of the labeled points is the graph of f concave up?

A, B

(e) At which of the labeled points does the concavity of the graph change from up to down?

C