Math 153 - Final Exam

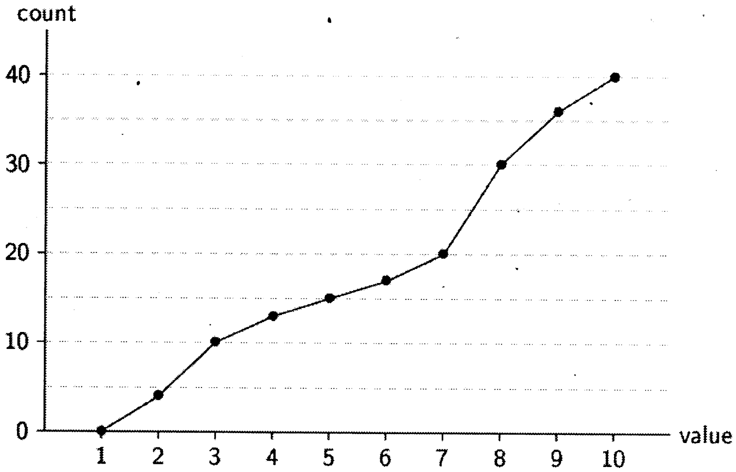
May 19, 2016

Name key

Score ____________

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) The graph below is an **ogive** showing the **cumulative frequencies** of certain recorded values.



(a) What is the total number of values in the sample?

$\boxed{40}$

(b) Over what range of values did the frequency increase the most?

THE RANGE FROM $\boxed{7 \text{ TO } 8}$

(c) About how many times did the value of 3 occur in the sample?

$10 - 4 = \boxed{6}$

2. (6 points) Test scores in Math 168 have a mean of 72.5 and a standard deviation of 4.6. Find the z-score associated with a score of 90. Is this score unusual? Explain.

$$Z = \frac{90 - 72.5}{4.6} \approx 3.804$$

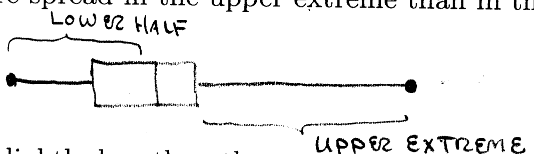
90 IS UNUSUALLY LARGE BECAUSE ITS Z-SCORE IS GREATER THAN 2.

3. (5 points) The mean of 30 test scores is 71.2. Two tests taken late had scores 58 and 67. Find the mean of all the test scores.

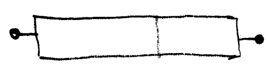
$$\bar{X} = \frac{30(71.2) + 58 + 67}{32} = \frac{2261}{32} \approx \boxed{70.66}$$

4. (6 points) For each part of this problem, sketch a boxplot that would correspond to a data set with the given characteristics.

(a) There is more spread in the upper extreme than in the lower half.



(b) The IQR is slightly less than the range.



5. (10 points) A jar contains 3 red marbles, 5 blue marbles, and 2 green marbles. A single marble is selected at random and its color is recorded.

(a) What is a possible sample space for this experiment?

$\{r, b, g\}$

(b) What is the probability of selecting a blue marble or a green marble?

$$P(\{b, g\}) = \frac{5+2}{10} = \boxed{\frac{7}{10}}$$

(c) Is your answer above a theoretical probability or an experimental probability?

(d) What is the probability of selecting a red marble given that the selected marble is not blue?

THERE ARE 3 REDS OUT OF 5 NOT BLUES → $\boxed{\frac{3}{5}}$

(e) What are the odds in favor of selecting a green marble?

2 green / 8 not green ⇒ ODDS ARE $\frac{2}{8}$ OR $\boxed{\frac{1}{4}}$

6. One hundred students at an American university were selected at random and asked to draw a line segment that measured 5 centimeters without using a ruler. The following frequency distribution summarizes the results.

Length of drawn segment (cm)	Frequency
1.2–2.4	6
2.5–3.7	10
3.8–5.0	28
5.1–6.3	43
6.4–7.6	7
7.7–8.9	6

(a) (3 points) What are the class boundaries associated with the last class listed above?

$\boxed{7.65 \text{ AND } 8.95}$

(b) (2 points) What is the class width?

$2.5 - 1.2 = \boxed{1.3}$

(c) (3 points) What are the class midpoints?

$\frac{2.4 + 1.2}{2} = \frac{3.6}{2} = 1.8$ $\boxed{1.8, 3.1, 4.4, 5.7, 7, 8.3}$

(d) (4 points) Use class midpoints to estimate the mean length of the segment drawn by the students.

$$\frac{1.8(6) + 3.1(10) + 4.4(28) + 5.7(43) + 7(7) + 8.3(6)}{100} = \boxed{5.089}$$

(e) (3 points) Use class midpoints to estimate the median.

$$\text{MEDIAN} = \frac{50^{TH} + 51^{ST}}{2} = \frac{5.7 + 5.7}{2} = \boxed{5.7}$$

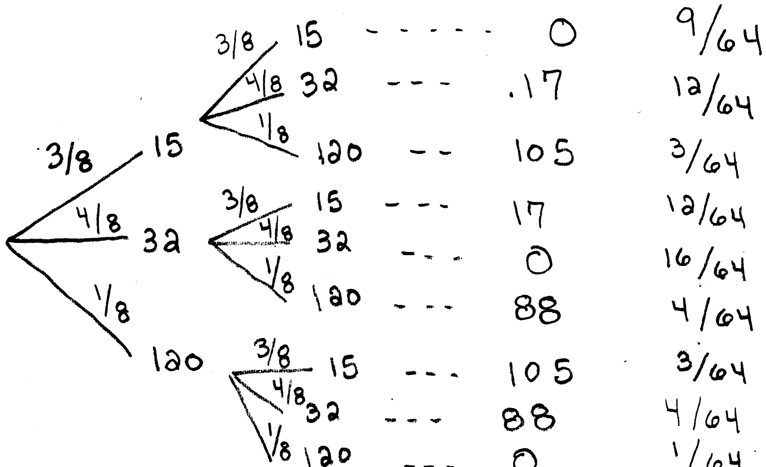
(f) (2 points) One of the students is selected at random. What is the probability that the student drew a line segment shorter than 6.35 cm?

$6 + 10 + 28 + 43$
$= 87$

PROB IS $\boxed{\frac{87}{100}}$

7. The local animal shelter currently has 3 dogs weighing 15 lbs, 4 dogs weighing 32 lbs, and 1 dog weighing 120 lbs. Two dogs are selected at random **with replacement** and their weights are recorded. Let the random variable x represent the range of the two-weight sample.

(a) (8 points) Sketch the probability tree associated with the selection of the two weights. Find the probability of each path.



(b) (2 points) What are the possible values of the random variable x?

0, 17, 88, 105

(c) (4 points) Determine the probability distribution for the random variable x. Give your distribution in the form of a table.

X	0	17	88	105
P(x)	$\frac{26}{64}$	$\frac{24}{64}$	$\frac{8}{64}$	$\frac{6}{64}$

(d) (4 points) Find the mean value of x.

$$\mu = \sum x\,P(x) = 0\left(\frac{26}{64}\right) + 17\left(\frac{24}{64}\right) + 88\left(\frac{8}{64}\right) + 105\left(\frac{6}{64}\right) \approx 27.2$$

(e) (4 points) Find the population range. Do the sample ranges target the population range? Explain. Did you expect this?

$120 - 15 = 105$

THE SAMPLE RANGES HAVE MEAN 27.2. SAMPLE RANGES DO NOT TARGET POP. RANGE. I EXPECTED THIS! WE LEARNED IN CLASS THAT THE RANGE IS A BIASED ESTIMATOR!

8. (12 points) A cereal manufacturer claims that the weight of a box of cereal labeled as weighing 12 ounces has mean weight 12.0 ounces and a standard deviation of 0.1 ounces. You collect and weigh a random sample of 75 boxes.

   (a) Based on the manufacturer's information, what is the probability that the mean weight of your sample is less than 11.99 ounces?

   $$P(\bar{x} < 11.99) = \text{normalcdf}(-999999, 11.99, 12, 0.1/\sqrt{75})$$

   $$\approx 0.1932$$

   (b) Suppose the mean weight of your sample turned out to be 11.97 ounces. If this were the case, would you believe that the manufacturer supplied accurate information? Explain.

   $$P(\bar{x} \leq 11.97) = \text{normalcdf}(-999999, 11.97, 12, 0.1/\sqrt{75})$$

   ≈ 0.004687 NO, THE PROB OF A MEAN WEIGHT THAT SMALL IS LESS THAN 0.5%

   (c) Based on the manufacturer's information, what is the cutoff weight for unusually small sample means.

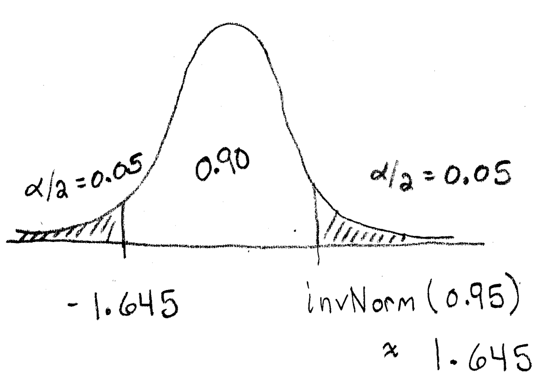
   $$12 - 2\left(\frac{0.1}{\sqrt{75}}\right) \approx 11.977$$

   (NOTICE THIS ADDS SUPPORT TO MY ANSWER FOR (b))

   (d) In order to trust your results above, must you assume the weights are normally distributed? Explain.

   NO, THE SAMPLE SIZE IS LARGE ENOUGH FOR THE SAMPLING DISTRIBUTION TO BE APPROXIMATELY NORMAL.

9. (6 points) Sketch the critical region associated with a two-tailed test for the mean, assuming the sampling distribution is normal (σ known) and $\alpha = 0.10$. Be sure to indicate the critical values.



10. (12 points) 57% of U.S. housing units are owner occupied. In a large community, 35 housing units are selected at random.

binomial
$n=35$
$p=0.57$
$q=0.43$

(a) What is the probability that at least 16 are owner occupied?

$$P(x \geq 16) = 1 - P(x < 16) = 1 - P(x \leq 15)$$
$$= 1 - \text{binomcdf}(35, 0.57, 15)$$
$$\approx 0.9350$$

(b) What is the probability that 20 or 21 are owner occupied?

$$P(x=20) + P(x=21) = \text{binompdf}(35, 0.57, 20) + \text{binompdf}(35, 0.57, 21) \approx 0.2633$$

(c) In the sample, what would be an unusually small number of owner-occupied housing units?

$$\mu - 2\sigma = np - 2\sqrt{npq} \approx 14.09$$

14 OR FEWER ARE UNUSUAL

(d) Since $P(x = 25) = 0.0313$, is it true that 25 is an unusually large number of owner-occupied housing units?

No, BECAUSE $P(x \geq 25) = 1 - \text{binomcdf}(35, 0.57, 24)$
$\approx 0.0581 > 0.05$

11. (5 points) Tax preparation charges by H&R Block, Inc. are approximately normally distributed with mean \$84.57 and standard deviation \$10.09. What charge is at the 85th percentile?

$$\text{invNorm}(0.85, 84.57, 10.09) \approx \$95.03$$

12. (6 points) Scientists want to estimate the mean weight of mice after they have been fed a special diet. From previous studies, it is known that the weights are normally distributed with standard deviation 3 grams. How many mice must be weighed so that a 95% confidence interval will have a margin of error of 0.5 grams?

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 \qquad n = \left(\frac{1.96 \cdot 3}{0.5}\right)^2 \approx 138.3$$

$$z_{\alpha/2} = \text{invNorm}(0.975) \approx 1.96$$

USE $n = 139$

13. (10 points) An airline has surveyed a simple random sample of air travelers to find out whether they would be interested in paying a higher fare in order to have internet access during their flight. Of the 400 travelers surveyed, 80 said internet access would be worth a slight extra cost. Find a 95% confidence interval estimate for the population proportion of air travelers who are in favor of the airline's internet idea. Give an interpretation of your interval in a complete sentence.

1-PropZInt

$X = 80$

$n = 400$

C-Level: 0.95

C.I. ESTIMATE IS (0.1608, 0.2392)

WE CAN BE 95% CONFIDENT THAT THE TRUE PROPORTION OF AIR TRAVELERS WHO SUPPORT THE IDEA IS BETWEEN 16% AND 24%.

14. (12 points) The mean age of passenger cars in the United States is 8.4 years. A random sample of 34 cars parked in the lot of a very large manufacturing plant gave a sample mean and sample standard deviation of 9.7 years and 3.1 years, respectively. We would like to conclude that the mean age of cars driven by the plant's employees is greater than the national average. Test the claim at the level $\alpha = 0.01$.

(a) State the null and alternative hypotheses.

CLAIM: $\mu > 8.4$

COUNTER: $\mu \leq 8.4$

$\longrightarrow$

$H_0 : \mu = 8.4$

$H_1 : \mu > 8.4$

(b) Compute the test statistic.

CALCULATOR'S T-Test

$t = 2.445$

(c) Find the P-value and draw a conclusion about the original claim.

P-VALUE $\approx$ 0.00999

THE P-VALUE IS VERY CLOSE TO α BUT SMALLER --- WE REJECT H_0.

THE EVIDENCE SUPPORTS THE CLAIM THAT $\mu > 8.4$ AT THE LEVEL $\alpha = 0.01$

15. (15 points) A brand-name antifungal ointment is known to deliver a mean of 3.5 micrograms of active ingredient to each square centimeter of skin. In testing a generic alternative, seven subjects apply the generic ointment. It is found that the absorbed amounts (in micrograms) of active ingredient are

2.6 3.2 2.1 3.0 3.1 2.9 3.7

(a) What is the mean amount of absorbed ingredient in the sample? What is the standard deviation?

CALCULATOR... $\bar{X} \approx 2.943$

$S \approx 0.4995$

(b) Use the level $\alpha = 0.01$ to test the claim that the mean amount absorbed from the generic ointment differs from that of the brand-name ointment.

T-Test w/ DATA

CLAIM: $\mu \neq 3.5 \Rightarrow H_0: \mu = 3.5$

$H_1: \mu \neq 3.5$

$\mu_0: 3.5$ $\quad t \approx -2.95$

$\longrightarrow$ P-VALUE ≈ 0.02558

P-VALUE IS NOT LESS THAN $\alpha \Rightarrow$ DO NOT REJECT H_0

(c) State the conclusions of your test in a complete sentence.

AT THE LEVEL $\alpha = 0.01$, THE EVIDENCE DOES NOT SUPPORT THE CLAIM THAT THE MEAN AMOUNT ABSORBED FROM GENERIC IS DIFFERENT FROM THE BRAND NAME.

(d) What assumption are you making if you expect your test to be valid?

BECAUSE THE SAMPLE IS SMALL $(7 < 30)$, WE MUST ASSUME THE POPULATION OF ABSORBED AMOUNTS IS NORMALLY DISTRIBUTED. (THEREFORE, THE T-TEST APPLIES.)