Math 153 - Test 3a

April 21, 2016

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

BINOMIAL
$n = 20$
$p = 0.223$
$q = 0.777$

1. (12 points) According to the U.S. Census Bureau, 22.3% of U.S. women aged 15 to 50 have had two children. Twenty American women in the 15–50 age group are selected at random.

   (a) What is the probability that exactly 6 of them have had two children?

   $P(x=6) = \text{binompdf}(20, 0.223, 6) \approx 0.1394$

   (b) What is the probably that at least 6 of them have had two children?

   $P(x \geq 6) = 1 - P(x<6) = 1 - P(x \leq 5)$

   $= 1 - \text{binomcdf}(20, 0.223, 5) \approx 0.2769$

   (c) How many women in the group of 20 should be expected to have had two children?

   $\mu = 20(0.223) = 4.46$

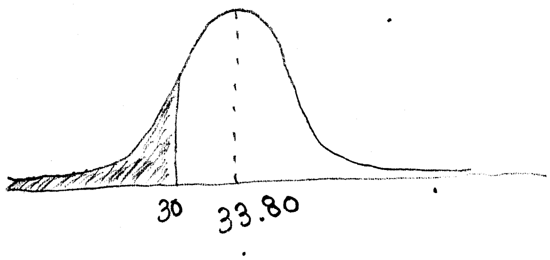
   (d) What would be an unusually large number of women in the sample to have had two children?

   $\sigma = \sqrt{npq} \approx 1.86$

   $4.46 + 2(1.86) \approx 8.18$

   9 or more

2. (6 points) Tarsus lengths of adult male grackles are normally distributed with mean 33.80 mm and standard deviation 4.84 mm. Roughly sketch the probability density curve associated with this distribution. Shade the area that corresponds to the probability $P(x < 30)$. Then compute the probability.

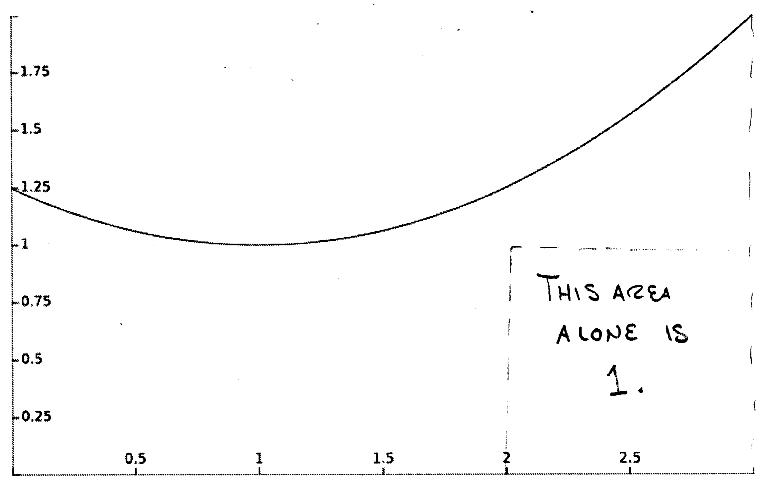


$P(x<30) =$

normalcdf(-99999, 30, 33.8, 4.84)

$\approx \boxed{0.2162}$

3. (3 points) Explain why the graph shown below cannot be a probability density curve.



THE AREA UNDER THE DENSITY CURVE IS GREATER THAN 1.

4. (3 points) A person draws 5 marbles, without replacement, from a jar containing a small number of only red marbles and blue marbles. Explain why this is definitely **not** a binomial process.

DRAWING WITHOUT REPLACEMENT MAKES THE FIVE TRIALS DEPENDENT AND CHANGES THE PROBABILITY OF SUCCESS AT EACH TRIAL.

5. (12 points) Telephone calls enter a university switchboard at an average rate of 38 calls per hour.

Poisson
$\mu = 38$

(a) In any given hour, what is the probability that the switchboard receives no more than 30 calls?

$$P(x \le 30) = \text{poissoncdf}(38, 30)$$

$$\approx \boxed{0.1089}$$

(b) In any given hour, what is the probability that the switchboard receives more than 40 calls?

$$P(x > 40) = 1 - P(x \le 40) = 1 - \text{poissoncdf}(38, 40)$$

$$\approx \boxed{0.3343}$$

(c) Use `poissonpdf` or `poissoncdf` (whichever is appropriate) to show that it is unusual for the switchboard to receive 26 calls in an hour.

$$P(x \le 26) = \text{poissoncdf}(38, 26) \approx 0.02587 < 0.05$$

$\Rightarrow$ 26 IS UNUSUALLY SMALL.

(d) How many calls should be expected in a week?

$$38 \times 7 \times 24 = \boxed{6384}$$

6. (3 points) A person draws 5 marbles, with replacement, from a jar containing red marbles, green marbles, and blue marbles. Explain why this is probably **not** a binomial process.

IT IS NOT CLEAR THAT THE OUTCOMES OF EACH TRIAL CAN BE CLASSIFIED SUCCESS OR FAILURE.

7. (15 points) The probability distribution for the random variable x is shown below.

x	0	1	2	3	4
$P(x)$	0.04	0.36	0.52	0.01	0.07

(a) What two things about the table above show that it is a probability distribution?

① $0 \le P(x) \le 1$ FOR EACH POSSIBLE X

② $\sum_i P(x) = 0.04 + 0.36 + 0.52 + 0.01 + 0.07 = 1$

(b) What is the mean value of x?

$$\mu = 0(0.04) + 1(0.36) + 2(0.52) + 3(0.01) + 4(0.07)$$

$$= \boxed{1.71}$$

(c) What is the standard deviation in x?

$$\sigma^2 = 0^2(0.04) + 1^2(0.36) + 2^2(0.52) + 3^2(0.01) + 4^2(0.07) - 1.71^2$$

$$= 0.7259$$

$$\Rightarrow \boxed{\sigma \approx 0.852}$$

(d) Use the mean and standard deviation to determine the unusual values of x.

$\mu - 2\sigma \approx 0.006$

$\mu + 2\sigma \approx 3.414$ $\Rightarrow$ $\boxed{\text{0 AND 4 ARE UNUSUAL}}$

(e) Use the 5% rule to determine the unusual values of x.

THE ONLY k FOR WHICH $P(x \le k) \le 0.05$ IS $\boxed{k = 0}$

THERE ARE NO k SUCH THAT $P(x \ge k) \le 0.05$.

8. (12 points) Birth weights of babies in Singapore are approximately normally distributed with mean 3135 grams and standard deviation 459 grams.

   (a) What is the probability that a randomly selected baby weighs more than 3500 grams?

   $P(x > 3500) = \text{normalcdf}(3500, 999999, 3135, 459)$

   ≈ 0.2132

   (b) What is the probability that a randomly selected baby weighs exactly 3135 grams?

   $P(x = 3135) = \boxed{0}$

   (c) What birth weight is at the 75th percentile?

   $\text{invNorm}(0.75, 3135, 459) \approx$ 3444.6 GRAMS

   (d) In a sample of 80 babies, about how many have birth weights between 3000 grams and 3200 grams?

   $80 \cdot P(3000 < x < 3200)$

   $= 80 \text{ normalcdf}(3000, 3200, 3135, 459)$

   ≈ 13.76

   ABOUT 14 BABIES

9. (3 points) Given the following discrete probability distribution, determine the value of $P(2 < x \leq 6)$.

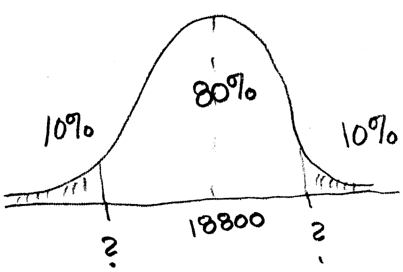
x	1	2	3	4	5	6	7	8	9
$P(x)$	0.01	0.12	0.07	0.22	0.08	0.35	0.04	0.01	0.10

$$P(x=3) + P(x=4) + P(x=5) + P(x=6)$$

$$= 0.07 + 0.22 + 0.08 + 0.35$$

$$= \boxed{0.72}$$

10. (8 points) An automobile dealer finds that used car prices are normally distributed with mean \$18,800 and standard deviation \$3240. The dealer decides to sell cars that appeal to the middle 80% of the market in terms of price (10% in each tail). Find the minimum and maximum prices of the cars the dealer will sell.



Low price = invNorm(0.10, 18800, 3240)

$= \boxed{\$\,14647.77}$

High price = invNorm(0.90, 18800, 3240)

$= \boxed{\$\,22952.23}$

11. (3 points) A computer program generates random real numbers that are uniformly distributed between 0 and 99. Sketch the probability density curve associated with the distribution of numbers.