Math 153 - Final Exam

May 16, 2013

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (12 points) In a probability experiment, the random variable x has six possible values: 0, 1, 2, 3, 4, 5. The (incomplete) probability distribution for x is shown below.

x	$P(x)$
0	0.013
1	0.032
2	0.044
3	0.271
4	0.468
5	??? ← 0.172

(a) Determine the missing probability, $P(5)$.

$$1 - (0.013 + 0.032 + 0.044 + 0.271 + 0.468)$$

$$= \underline{0.172}$$

(b) Compute the mean value of the random variable.

$\underline{\mu = 3.665}$ (CALCULATOR)

(c) Compute the standard deviation in the values of x.

$\sigma = 1.0013865\ldots$

(CALCULATOR)

$\underline{\sigma \approx 1.00}$

(d) Determine the unusually small and large values of x.

0 & 1 ARE UNUSUALLY SMALL BECAUSE $P(x \le 1) < 5\%$

THERE ARE NO UNUSUALLY LARGE VALUES.

2. (10 points) The diameters of Douglas firs grown for Christmas trees are approximately normally distributed with mean 4 in and standard deviation 1.5 in.

   (a) About what percent of trees have diameters between 3 and 5 inches?

   normalcdf(3, 5, 4, 1.5) $\approx$ $\underline{0.495}$

   (b) About what percent of trees have diameters greater than 6 in?

   normalcdf(6, 99999, 4, 1.5) $\approx$ $\underline{0.0912}$

   (c) In a group of 500 trees, about how many have diameters greater than 6 in?

   $500 \times 0.0912 \approx \underline{\underline{46}}$

3. (10 points) According to the *Humane Society of the United States*, 40% of all U.S. households own at least one dog. A sample of 15 U.S. households is selected at random.

   (a) What is the probability that exactly 8 of the households have at least one dog?

   $P(x=8)$ = binomialpdf(15, 0.40, 8) $\approx$ $\underline{0.118}$

   (b) What is the probability that at least 6 households have at least one dog?

   $P(x \geq 6) = 1 - P(x<6) = 1 - P(x \leq 5)$

   1 − binomialcdf(15, 0.40, 5) $\approx$ $\underline{0.597}$

   (c) In the sample of size 15, what would be an usually large number of households that have at least one dog?

   $$\mu + 2\sigma = np + 2\sqrt{npq}$$
   $$= 15(0.40) + 2\sqrt{(15)(0.40)(0.60)}$$
   $$\approx 9.79 \approx \underline{\underline{10}}$$

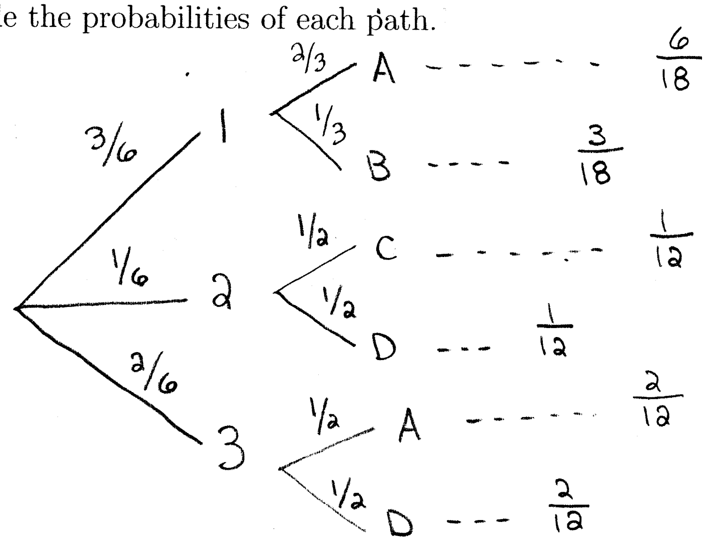
4. (14 points) A number is randomly selected from the following box of numbers.

1	1	1	2	3	3

Next, a letter is selected at random from the box corresponding to your number.

Box #1	Box #2	Box #3
A A B	C D	A D

(a) Sketch the complete tree diagram associated with this two-stage experiment. Include the probabilities of each path.



(b) What is the probability of selecting the letter A?

$$\frac{6}{18} + \frac{2}{12} = \frac{1}{2}$$

(c) What are the odds against selecting the letter A?

$$\frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{1}$$

(d) What is the probability of selecting the letter A or the letter C?

$$\frac{1}{2} + \frac{1}{12} = \frac{7}{12}$$

5. (8 points) For cars traveling at 30 miles per hour (mph), the distance required to brake to a stop is normally distributed with mean 50 ft and standard deviation 8 ft.

   (a) What would be considered an unusually large stopping distance?

   $50 + 2(8) = \underline{66 \text{ FT}}$

   (b) Within what distance can 90% of all cars brake to a stop?

   invNormal $(0.90, 50, 8) \approx \underline{60.25 \text{ FT}}$

6. (16 points) A survey of 60 adult men found that their mean height was 69 in. Assume that the standard deviation of the population is 3.5 in.

   (a) Is the underlying sampling distribution normal or t? How do you know?

   NORMAL, BECAUSE THE POP. STD. DEV IS KNOWN (OR $N > 30$)

   (b) Construct a 95% confidence interval estimate for the mean height of adult men.

   ZInterval
   $\sigma = 3.5$
   $\bar{X} = 69$
   $n = 60$

   95% C.I. IS

   $(68.114, 69.886)$

   (c) Determine the sample size required to have a margin of error of ± 0.25 at the level $\alpha = 0.01$.

   $\alpha = 0.01 \Rightarrow \alpha/2 = 0.005 \Rightarrow z_{\alpha/2} = \text{invNormal}(0.995) \approx 2.576$

   $n = \left(\frac{2.576 \cdot 3.5}{0.25} \right)^2 \approx \underline{1301}$

   (d) Suppose the population standard deviation was not known, but the sample size remained 60. Would the underlying sampling distribution be normal or t? Explain.

   STILL NORMAL BECAUSE $N = 60 > 30$.

7. (10 points) Suppose college professors at two-year institutions earn an average of \$65,608 per year with a standard deviation of \$4000. A sample of 100 two-year-college professors is randomly selected.

(a) What is the probability the the sample mean is greater than \$67,000?

$$\text{normalcdf}\left(67000, 999999, 65608, \frac{4000}{\sqrt{100}}\right)$$

$$\approx \underline{0.000251}$$

(b) If your random sample actually produced a sample mean of \$67,000, would you consider that unusual? Explain.

YES, VERY UNUSUAL, BECAUSE THE PROBABILITY IN PART (a) IS MUCH LESS THAN 5%.

(c) If the sample size was increased to 400, what would happen to your probability in part (a)? Why?

IT WOULD GET MUCH SMALLER. AS THE SAMPLE SIZE INCREASES, THE SPREAD IN THE SAMPLING DISTRIBUTION DECREASES.

8. (10 points) The five-number summary for a data set is:

Min = 14, $Q_1 = 37$, Med = 41, $Q_3 = 43$, Max = 50

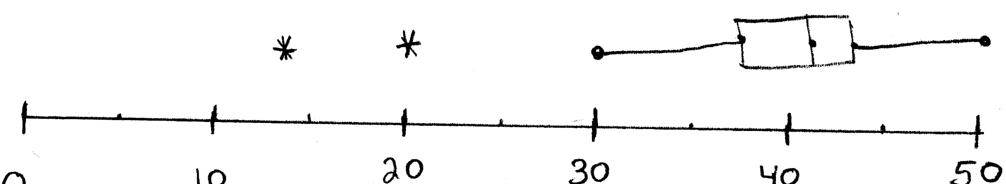
Compute the cut-off values for outliers. Then construct an example of a modified boxplot with the given five-number summary.

$IQR = 43 - 37 = 6$

$1.5 \times IQR = 9$

$Q_1 - 1.5 \times IQR = \underline{28}$ ← 14 IS AN OUTLIER

$Q_3 + 1.5 \times IQR = \underline{52}$ ← NO OUTLIERS ON UPPER END



9. (16 points) It has been found that harvested avocados quickly become infested with fruit flies. The table below shows the percent of infested avocados (y) in a large harvest x days after they were harvested.

Days after Harvest, x	Precent Infested, y
1	30
2	40
4	45
5	57
6	60
7	75
9	100

(a) Compute the linear correlation coefficient, r, and use it to draw a conclusion about the strength of the linear relationship between x and y.

$r \approx 0.96925$

SINCE THE IS VERY CLOSE TO +1, WE CONCLUDE THAT THERE IS A STRONG POSITIVE LINEAR CORRELATION.

(b) Compute the corresponding P-value and draw a conclusion about the existence of a linear relationship.

$P \approx 0.0003133$

SINCE THIS P-VALUE IS VERY SMALL, IT REINFORCES THE CONCLUSION ABOVE.

(c) Find the regression equation and use it to predict the percent of infestation 8 days after harvest.

$\hat{y} = 18.4268 + 8.1768x$

WHEN $X = 8$, $\hat{y} \approx 83.8\%$

(d) Use your regression equation to predict how long ago the avocados were harvested if 85% are infested.

$85 = 18.4268 + 8.1768x$

$\Rightarrow x \approx 8.14$ DAYS

10. (24 points) A new process has been developed to produce synthetic diamonds. Six synthetic diamonds are randomly selected from a large batch that were produced by the new process. Their weights, in karats, are given below.

0.61, 0.52, 0.48, 0.57, 0.54, 0.46

(a) Find the sample mean and sample standard deviation.

$\bar{X} = 0.53$ $\quad\quad$ $S = 0.05586$

(b) Using your sample statistics, determine the weights of unusually small and large diamonds.

$\bar{X} - 2S \approx 0.418$

$\bar{X} + 2S \approx 0.642$

(c) Find a 95% confidence interval estimate for the mean weight of diamonds produced by the new process.

TInterval

USING DATA ABOVE

95% C.I. IS

$(0.47138, 0.58862)$

(d) The developers of the process claim that it produces diamonds that weigh more than 0.5 karats, on average. You wish to test the developer's claim. What are your null and alternative hypotheses?

CLAIM : $\mu > 0.5$

CONTRARY CLAIM : $\mu \leq 0.5$

$H_0 : \mu = 0.5$

$H_1 : \mu > 0.5$

(e) Test the developer's claim in part (d) at the level $\alpha = 0.05$. Find the P-value and state your conclusion.

T-Test

USING DATA ABOVE

$P \approx 0.1227 > 0.05 \Rightarrow$ DO NOT REJECT H_0

THE EVIDENCE DOES NOT SUPPORT THE DEVELOPER'S CLAIM.

(f) How does your confidence interval in part (c) support your conclusion in part (e)?

SINCE 0.5 IS CONTAINED IN THE C.I., WE CANNOT REJECT THE NULL HYPOTHESIS.

11. (10 points) In the United States from 1850 to 2000, there were an average of 17.93 hurricanes per decade.

   (a) In any given decade, what is the probability that 13 or fewer hurricanes affect the United States?

   $P(x \le 13) = \text{poissoncdf}(17.93, 13) \approx \underline{0.146}$

   (b) In any given decade, what is the probably that there are exactly 16 hurricanes?

   $P(x=16) = \text{poissonpdf}(17.93, 16) \approx \underline{0.0891}$

   (c) In the 1880s, the US was hit by 27 hurricanes. Is this an unusually large number of hurricanes in a decade?

   $\mu + 2\sigma = 17.93 + 2\sqrt{17.93} \approx 26.4$

   YES, 27 IS UNUSUALLY LARGE.

12. (10 points) Suppose A and B are events such that $P(A) = 0.48$, $P(B) = 0.55$, and $P(A \cap B) = 0.264$.

   (a) Compute $P(\overline{A})$.

   $1 - 0.48 = \underline{0.52}$

   (b) Compute $P(A \cup B)$.

   $P(A) + P(B) - P(A \cap B) = 0.48 + 0.55 - 0.264$

   $= \underline{0.766}$

   (c) Compute $P(A|B)$.

   $\frac{P(A \cap B)}{P(B)} = \frac{0.264}{0.55} = 0.48$

   (d) Are A and B independent? Explain.

   YES, BECAUSE $P(A) = P(A|B)$.

   (e) What are the odds in favor of B?

   $\frac{0.55}{0.45} = \frac{55}{45} = \underline{\frac{11}{9}}$