Math 153 - Test 2a

March 21, 2013

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) The upper leg lengths, in centimeters, of 20 adult females are shown below.

27.0	31.1	33.2	36.0	36.2	38.1	38.2	38.2	38.7	39.0
39.0	39.7	39.9	40.2	40.3	41.5	41.6	41.6	43.8	48.6

Determine the quartiles, the interquartile range, and the cutoff values for outliers. Then sketch the modified boxplot.

$Q_1 = \frac{36.2 + 38.1}{2} = 37.15$

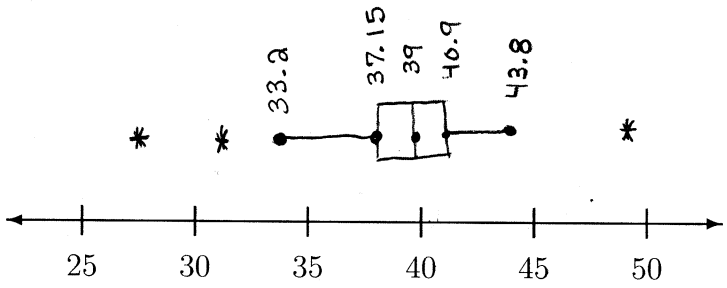
$\text{MED} = \frac{39 + 39}{2} = 39$

$Q_3 = \frac{40.3 + 41.5}{2} = 40.9$

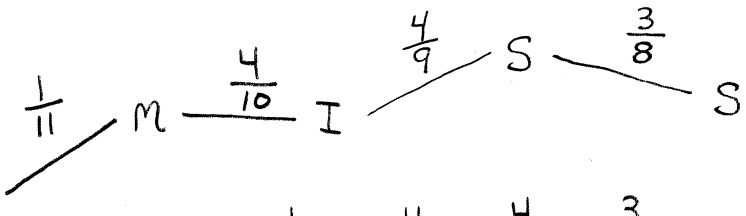
$IQR = 40.9 - 37.15 = 3.75$

$Q_1 - 1.5 \times IQR = 31.525$

$Q_3 + 1.5 \times IQR = 46.525$



2. (6 points) Four letters are selected at random, without replacement, from the word *MISSISSIPPI*. What is the probability of spelling *MISS*?



$\frac{1}{11} \times \frac{4}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{48}{7920} \approx 0.6\%$

3. (10 points) A number and a letter are simultaneously selected at random from each box.

1	2	3

A	A	B

(a) What is the sample space for this experiment?

$\{(1,A), (1,B), (2,A), (2,B), (3,A), (3,B)\}$

(b) Is your sample space uniform (equally likely)? Explain.

No, since there are 2 A's, then selections with A's are more likely than the selections with B's.

(c) What is the event of selecting the letter B from the second box?

$\{(1,B), (2,B), (3,B)\}$

(d) Let Y be the event of selecting an odd number from the first box. What is $\overline{Y}$?

$\overline{Y} = \{(2,A), (2,B)\}$

(e) With Y as defined above, let Z be the event of selecting the letter A from the second box. What is $Y \cap Z$?

$Y \cap Z = \{(1,A), (3,A)\}$

4. (6 points) The five-number summary for a data set is:

$$\text{Min} = 14, \quad Q_1 = 37, \quad \text{Med} = 41, \quad Q_3 = 43, \quad \text{Max} = 50$$

What are the cutoff values for outliers? Does the data set have any outliers?

IQR = 43 - 37 = 6

$37 - 1.5 \times 6 = 28$ } CUTOFF VALUES

$43 + 1.5 \times 6 = 52$ }

YES! SINCE THE MIN VALUE IS LESS THAN THE LOWER CUTOFF, THERE ARE OUTLIERS ON THE LOW END.

5. (7 points) A letter is selected at random from each box.

A B C C	A A C	A A A B C	A B

What is the probability that at least one letter A is selected?

PROB OF "NO A'S" = $\frac{3}{4} \times \frac{1}{3} \times \frac{2}{5} \times \frac{1}{2} = \frac{1}{20}$

PROB OF NOT "NO A'S" = PROB OF AT LEAST ONE A

$= 1 - \frac{1}{20} = \boxed{\frac{19}{20}}$

6. (6 points) A card is drawn at random from a standard deck of playing cards. Let A be the event of drawing a face card, and let B be the event of drawing a queen. Compute $P(A|B)$ and $P(B|A)$. Be sure to indicate which is which.

P(A|B) = PROB OF FACE CARD GIVEN QUEEN = 1 SINCE EVERY QUEEN IS A FACE CARD

P(B|A) = PROB OF QUEEN GIVEN FACE CARD

$= \frac{4}{12} = \frac{1}{3}$ SINCE THERE ARE 4 QUEENS OUT OF THE 12 FACE CARDS

7. (10 points) Suppose A and B are events such that $P(\overline{A}) = 0.25$, $P(B) = 0.5$, and $P(A \cup B) = 0.87$.

(a) Determine $P(A)$.

$P(A) = 1 - 0.25 = \boxed{0.75}$

(b) Determine $P(A \cap B)$.

$0.87 = 0.75 + 0.5 - P(A \cap B)$

$\Rightarrow P(A \cap B) = \boxed{0.38}$

(c) Find the odds against B.

$\frac{P(\overline{B})}{P(B)} = \frac{0.5}{0.5} = \boxed{\frac{1}{1}}$

(d) Determine $P(A|B)$.

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.38}{0.5} = \boxed{0.76}$

(e) Are A and B independent? Explain.

No, because $P(A|B) = 0.76 \neq P(A) = 0.75$

8. (9 points) A teacher collected the following data on her students.

	Completed assignments	Did not complete assignments
Passed class	217	59
Failed class	18	67

← 361 TOTAL STUDENTS

A student is selected at random.

(a) What is the probability that the student completed assignments?

$$\frac{217+18}{361} = \boxed{\frac{235}{361}}$$

(b) What is the probability that the student failed the class?

$$\frac{18+67}{361} = \boxed{\frac{85}{361}}$$

(c) What is the probability that the student completed assignments and passed the class?

$$\boxed{\frac{217}{361}}$$

(d) What is the probability that the student passed the class given that the student completed assignments?

$$\frac{217}{217+18} = \boxed{\frac{217}{235}}$$

(e) Are passing the class and completing assignments independent of one another? Show work or explain.

PROB OF PASSING CLASS $= \frac{217+59}{361} = \frac{276}{361} \approx 76.5\%$

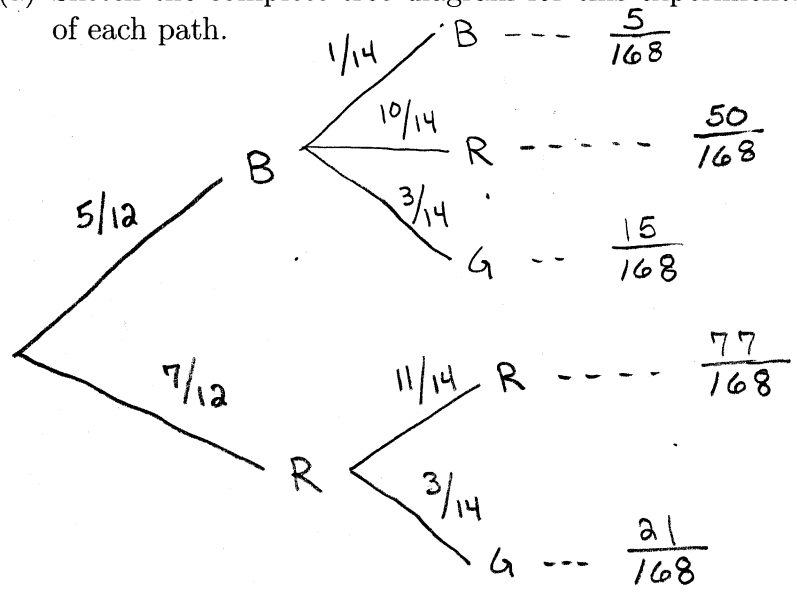
PROB OF PASSING GIVEN COMPLETED ASSIGNMENTS $= \frac{217}{235}$

$\approx 92.3\%$

No! $76.5\% \neq 92.3\%$

9. (12 points) Box One contains 5 blue marbles and 7 red marbles. Box Two contains 10 red marbles and 3 green marbles. A marble is selected at random from Box One and placed into Box Two. Then a marble is selected at random from Box Two.

(a) Sketch the complete tree diagram for this experiment. Include the probabilities of each path.



(b) What are the odds in favor of selecting a blue marble from Box Two?

$$\frac{\frac{5}{168}}{\frac{163}{168}} = \boxed{\frac{5}{163}}$$

(c) What is the probability of selecting a blue marble from Box One or a red marble from Box Two?

$$\frac{5}{12} + \frac{77}{168} = \boxed{\frac{147}{168}}$$

(d) What is the probability of selecting a blue marble from Box Two given that a red marble was selected from Box One?

Impossible. Prob is zero.

10. (9 points) Three letters are selected, without replacement, from the English alphabet. Let x represent the number of vowels that are selected. The probability distribution for x is shown below.

x	$P(x)$	
0	$\frac{7980}{15600}$	$\approx$ 51%
1	$\frac{6300}{15600}$	$\approx$ 40%
2	$\frac{1260}{15600}$	$\approx$ 8%
3	$\frac{60}{15600}$	$\approx$ 0.4%

(a) How can you be certain that the table describes a probability distribution?

ALL PROBABILITIES ARE BETWEEN 0 AND 1

- AND -

$$\frac{7980 + 6300 + 1260 + 60}{15600} = \frac{15600}{15600} = 1$$

(b) What is the expected value of x?

$$\bar{X} = 0\left(\frac{7980}{15600}\right) + 1\left(\frac{6300}{15600}\right) + 2\left(\frac{1260}{15600}\right) + 3\left(\frac{60}{15600}\right)$$

$$= \boxed{\frac{9000}{15600} \approx 0.577}$$

(c) Are there any unusually large or small values of x?

SINCE $P(x \geq 3) \approx 0.4\% < 5\%$

3 IS UNUSUALLY LARGE

Math 153 - Test 2b

March 21, 2013

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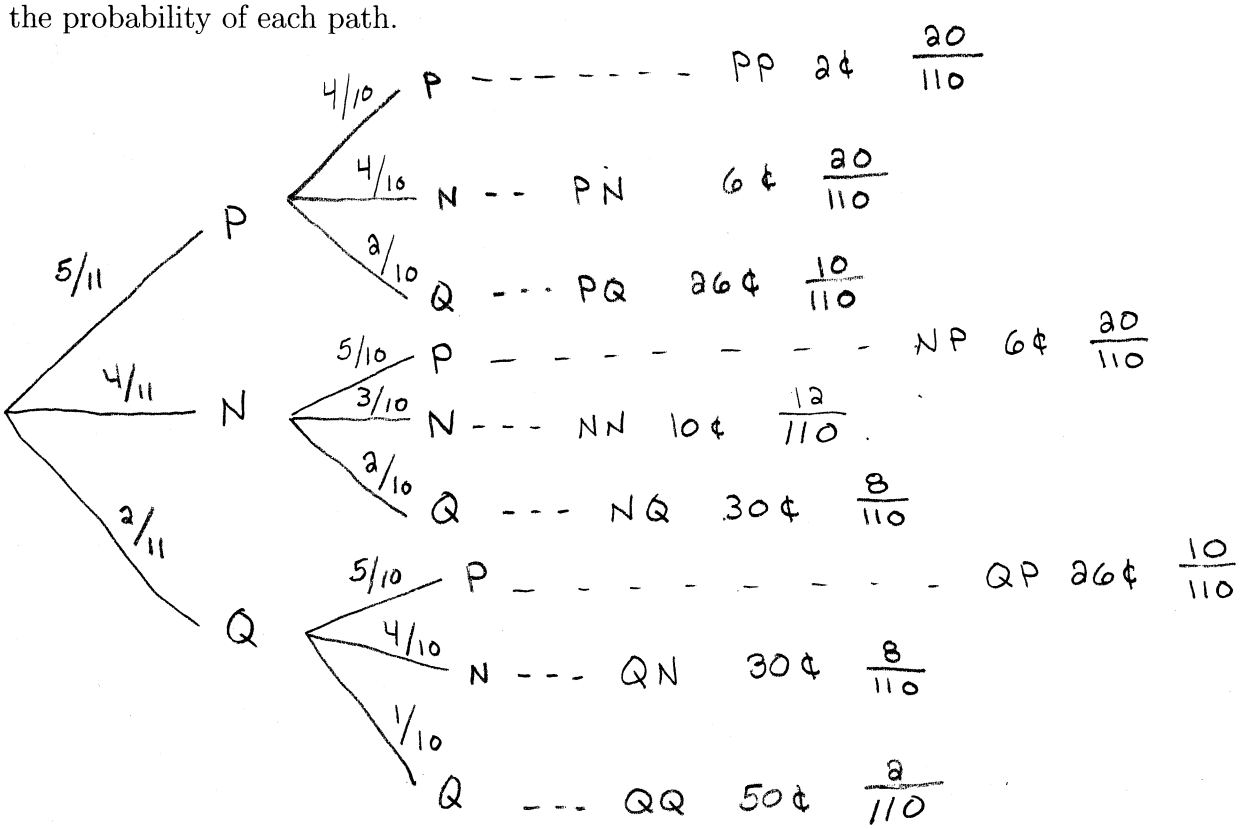
Show all work to receive full credit. Supply explanations where necessary. This portion of the test is worth 15 points and is due Monday, April 2. You must work individually on this test.

Two coins are selected at random (without replacement) from a jar containing 5 pennies, 4 nickels, and 2 quarters. Let x represent the total value, in cents, of the selected coins.

1. Thinking about this procedure as a two-stage experiment, sketch the probability tree. Include the probability of each path.



2. What are all the possible values of the random variable x.

2, 6, 26, 10, 30, 50

3. Determine the probability distribution for x.

X	$P(x)$
2	$20/110$
6	$40/110$
10	$12/110$
26	$20/110$
30	$16/110$
50	$2/110$

4. Find the mean (expected value) for x.

$$\bar{X} = 2\left(\frac{20}{110}\right) + 6\left(\frac{40}{110}\right) + 10\left(\frac{12}{110}\right) + 26\left(\frac{20}{110}\right) + 30\left(\frac{16}{110}\right) + 50\left(\frac{2}{110}\right)$$

$$= \frac{1500}{110} = 13.\overline{63} \approx \boxed{13.6\ ¢}$$

5. Find the variance and standard deviation.

$$\sigma^2 = 4\left(\frac{20}{110}\right) + 36\left(\frac{40}{110}\right) + 100\left(\frac{12}{110}\right) + 676\left(\frac{20}{110}\right) + 900\left(\frac{16}{110}\right) + 2500\left(\frac{2}{110}\right) - \left(\frac{1500}{110}\right)^2$$

$$= \frac{16704}{121} \approx \boxed{138.05}$$

$$\sigma = \frac{\sqrt{16704}}{11} \approx \boxed{11.75\ ¢}$$

6. Are any of the possible values for x unusually small or large? Explain.

$\bar{X} - 2\sigma \approx -9.9\ ¢$

$\bar{X} + 2\sigma \approx 37.1\ ¢ \longrightarrow X = 50\ ¢$ IS UNUSUALLY LARGE.