Math 151 - Test 2

October 20, 2015

Name key

Score

Show all work. Supply explanations where necessary.

1. (7 points) Let $f(x) = x^2 + 1$ and $g(x) = x + 4$.

   (a) Find and simplify a formula for $(f \circ g)(x)$.

   $$f(g(x)) = (x+4)^2 + 1 = \boxed{x^2 + 8x + 17}$$

   (b) Evaluate $(f \circ g)(-2)$.

   $$f(g(-2)) = f(2) = \boxed{5}$$

   (c) Evaluate $(g \circ f)(4)$.

   $$g(f(4)) = g(17) = \boxed{21}$$

2. (10 points) Find the inverse of the function $g(x) = \sqrt{x+3}$ and sketch the graphs of both g and g^{-1}. Label which is which.

Domain of $g = [-3, \infty)$

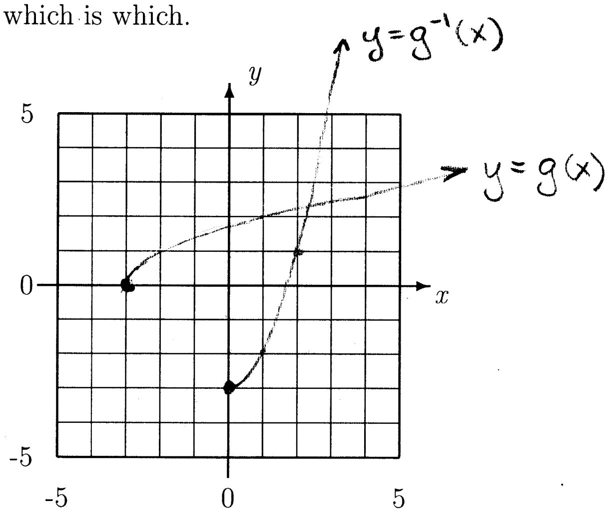
$y = \sqrt{x+3}$

$y^2 = x + 3$

$y^2 - 3 = x$

$y = x^2 - 3$

Range of $g = [0, \infty)$



$$\boxed{g^{-1}(x) = x^2 - 3, \quad x \geq 0}$$

3. (4 points) Find two functions f and g so that $(f \circ g)(x) = (3x+2)^5 - (3x+2)^2$.

INSIDE: $g(x) = 3x + 2$

OUTSIDE: $f(x) = x^5 - x^2$

THESE MAKE

$f(g(x)) =$

4. (2 points) Some values of the function h are given in the table below. Assuming that h has an inverse function, use the data in the table to determine $h^{-1}(8)$.

x	0	2	4	6	8
$h(x)$	1	2	5	8	7

BECAUSE $h(6) = 8$,

$h^{-1}(8) = 6$

5. (8 points) Find the exact values of the real and complex zeros of $H(x) = 2x^4 - 11x^3 + 20x^2$. Show all work.

$$H(x) = x^2(2x^2 - 11x + 20)$$

ZEROS OF x^2 ARE

$x = 0,\ x = 0$

ZEROS...

$$\frac{11 \pm \sqrt{121 - 4(2)(20)}}{4} = \frac{11 \pm \sqrt{-39}}{4} = \frac{11 \pm \sqrt{39}\,i}{4}$$

ZEROS ARE

$x = 0,\ x = 0,$

$x = \dfrac{11 + \sqrt{39}\,i}{4},$

$x = \dfrac{11 - \sqrt{39}\,i}{4}$

6. (6 points) Use synthetic division to determine $Q(-5)$ when

$$Q(x) = 3x^4 - 5x^3 + 12x^2 - 7x + 3.$$

$$\begin{array}{r|rrrrr} -5 & 3 & -5 & 12 & -7 & 3 \\ & & -15 & 100 & -560 & 2835 \\ \hline & 3 & -20 & 112 & -567 & 2838 \end{array}$$

$Q(-5) = 2838$

7. (2 points) True or False: If $3 - 5i$ is a zero of a 2nd degree polynomial with real coefficients, then $-3 + 5i$ is also a zero.

FALSE, $3 + 5i$ IS ALSO A ZERO,

NOT $-3 + 5i$

8. (12 points) Consider the polynomial $p(x) = 6x^3 - 13x^2 - 4x + 1$.

(a) List the possible rational zeros of $p(x)$. 1, 2, 3, 6   1

$\pm \left\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{6} \right\}$

(b) The polynomial $p(x)$ has one rational zero and two irrational zeros. Find the exact values of the zeros. Show all work.

$\sqrt{8} = 2\sqrt{2}$

OF THE POSSIBLE RATIONAL ZEROS,

FROM THE CALCULATOR, IT LOOKS LIKE $X = \frac{1}{6}$

LET'S CHECK...

$$\begin{array}{r|rrrr} \frac{1}{6} & 6 & -13 & -4 & 1 \\ & & 1 & -2 & -1 \\ \hline & 6 & -12 & -6 & 0 \end{array}$$

$\left(x - \frac{1}{6}\right)\left(6x^2 - 12x - 6\right)$

$= \left(x - \frac{1}{6}\right)(6)\left(x^2 - 2x - 1\right)$

QF...

$$\frac{2 \pm \sqrt{4 - 4(1)(-1)}}{2} = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

ZEROS ARE

$X = \frac{1}{6}$, $X = 1 + \sqrt{2}$,

$X = 1 - \sqrt{2}$

9. (12 points) Consider the polynomial $f(x) = 2(x-3)^2(x+1)^3$.

(a) Determine the degree of f and the leading coefficient.

$\text{Deg}(f) = 5$

LEADING COEFF $= 2(1)^2(1)^3 = 2$

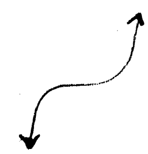
(b) State the zeros of f and their corresponding multiplicities.

$x = 3$ MULTIPLICITY 2 (BOUNCES / FLATTENS)

$x = -1$ MULTIPLICITY 3 (CROSSES / FLATTENS)

(c) Describe the end behavior of the graph of f. (A picture or diagram will work!)

SAME AS $y = 2x^5$ $\longrightarrow$

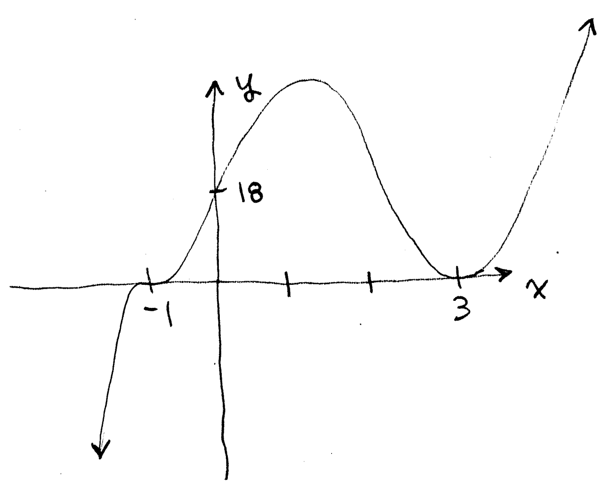


(d) Determine the y-intercept.

$f(0) = 2(-3)^2(1)^3 = 18$

$\Rightarrow \boxed{(0, 18)}$

(e) Roughly sketch the graph of f. Be sure that your graph correctly illustrates the y-intercept, the end behavior, and the behavior near the x-intercepts.

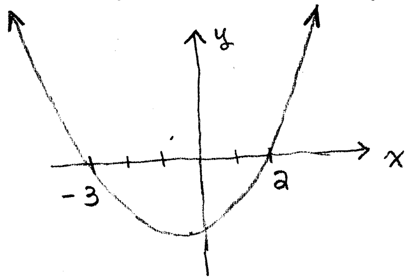


10. (8 points) Solve the polynomial inequality $x^2 + x \leq 6$. Write your solution in interval notation.

$x^2 + x - 6 \leq 0$

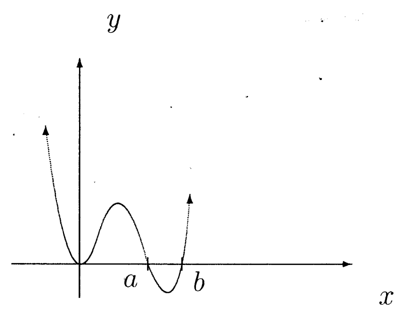
$(x+3)(x-2) \leq 0$

$x = -3 \; , \; x = 2$



GRAPH BELOW AXIS ON $\boxed{[-3, 2]}$

11. (6 points) Which polynomial below has the given graph? State at least two reasons for your choice.



(a) $p(x) = x(x-a)(x-b)$

(b) $p(x) = (x+a)(x+b)$

(c) $p(x) = x^2(x-a)(x-b)$

(d) $p(x) = x(x-a)^2(x-b)^2$

① BOUNCES AT $X = 0$

② CROSSES AT $X = a, \; X = b$

12. (3 points) Use the graph above (problem #11) to solve the inequality $p(x) > 0$. Write your solution in interval notation.

$p(x) > 0$ ON $\boxed{(-\infty, 0) \cup (0, a) \cup (b, \infty)}$

13. (10 points) Use polynomial long division to compute the quotient and remainder when $x^4 - 3x^3 + 8x - 6$ is divided by $x^2 + 2x$. Write your answer in the form $q(x) + r(x)/d(x)$.

$$
\begin{array}{r}
x^2 - 5x + 10 \\
x^2 + 2x \,\big)\, x^4 - 3x^3 + 0x^2 + 8x - 6 \\
-(x^4 + 2x^3) \\
\hline
-5x^3 + 0x^2 + 8x - 6 \\
-(-5x^3 - 10x^2) \\
\hline
10x^2 + 8x - 6 \\
-(10x^2 + 20x) \\
\hline
-12x - 6
\end{array}
$$

$$\boxed{x^2 - 5x + 10 + \frac{-12x-6}{x^2+2x}}$$

14. (10 points) Consider the polynomial $P(x) = 2x^3 - x^2 - 13x - 6$.

1, 2   1, 2, 3, 6

(a) List all possible rational zeros.

$\pm\{1, \frac{1}{2}, 2, 3, \frac{3}{2}, 6\}$

(b) Use your graphing calculator to determine which, if any, of the possible rational zeros are actual zeros.

Looks like $-2, -\frac{1}{2}, 3$.

I'll check by plugging in.

$P(-2) = 2(-2)^3 - (-2)^2 - 13(-2) - 6$
$= -16 - 4 + 26 - 6 = 0$ ✓

$P(3) = 2(3)^3 - (3)^2 - 13(3) - 6$
$= 54 - 9 - 39 - 6 = 0$ ✓

$P(-\frac{1}{2}) = 2(-\frac{1}{2})^3 - (-\frac{1}{2})^2 - 13(-\frac{1}{2}) - 6$
$= -\frac{2}{8} - \frac{1}{4} + \frac{13}{2} - 6 = 0$ ✓

(c) Use your zeros to determine the complete factorization of P.

Linear factors are

$(x+2)(x+\frac{1}{2})(x-3)$

Mult by 2 to get correct leading coefficient

$$\boxed{P(x) = 2(x+2)(x+\tfrac{1}{2})(x-3)}$$