

Math 132 - Test 3

November 16, 2022

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. You must work individually on this test. Please do not confuse **sequences** and **series**. The test is due November 28, 2022 at 12:30 pm.

1. (5 points) Use the data given below and the trapezoid rule with $n = 8$ to approximate

$$\int_0^2 f(x) dx.$$

x	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
$f(x)$	4.27	4.38	4.61	5.84	6.12	7.24	7.60	8.15	8.28

2. (5 points) Use Simpson's rule with $n = 6$ to approximate $\int_1^3 \sin \sqrt{x} dx$.

3. (5 points) Consider the definite integral $\int_2^3 (-x^2 + 5x - 6) dx$.

(a) Thinking about the graph of the integrand, would you expect the trapezoid rule to underestimate or overestimate the value of the integral. Explain.

(b) What about Simpson's rule?

4. (15 points) Consider the integral $\int_0^\infty \frac{dx}{\sqrt{x}(x+1)}$.

(a) Explain why/how this integral is improper.

(b) Evaluate the indefinite integral $\int \frac{dx}{\sqrt{x}(x+1)}$. (Hint: Use $u = \sqrt{x}$.)

(c) Write the original improper integral with limits and evaluate.

5. (5 points) Consider the sequence whose n th term is $a_n = n \sin\left(\frac{1}{n}\right)$. Determine whether the sequence converges or diverges. If it converges, find its limit.
6. (10 points) For each part of this problem you are asked to give an example of a sequence with a certain property. If it is not obvious that your sequence satisfies the property, be sure to explain. If it is not possible to give such an example, explain why.
- (a) Give an example of a divergent sequence with a convergent subsequence.
- (b) Give an example of a nonconstant sequence that has limit $3/4$.
- (c) Give an example of an unbounded sequence that converges with limit 100.
- (d) Give an example of a sequence that is recursively defined.

7. (5 points) Determine whether the series converges or diverges. If it converges, find its sum.

$$\sum_{n=1}^{\infty} \left(\frac{3^n + (-2)^n}{5^n} \right)$$

8. (5 points) The following series converges. Find its sum.

$$\sum_{n=1}^{\infty} \frac{1}{9n^2 + 3n - 2}$$

9. (10 points) Consider the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(n+1)}$.

(a) Use the result of an earlier problem on this test to show that the series converges.

(b) By using an appropriate p -series, use limit comparison to show that the series converges.

10. (5 points) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$ converges or diverges. If it converges, does it converge absolutely or conditionally?

11. (5 points) Explain why the alternating series test does not apply to the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n \cos(\pi n)}$. Does the series converge or diverge?

12. (5 points) We will soon be able to show that $\sin(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$.

(a) Compute S_4 , the partial sum through $n = 4$. Carry enough decimal digits through your computations so that you are reasonably sure that your final answer has at least 8 correct digits.

(b) Use the alternating series remainder theorem to determine an upper bound on the error made in the approximation $\sin(1) \approx S_4$.

13. (20 points) Determine whether the series converges or diverges. Show all work when applying our tests.

(a)
$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 + 1}$$

(b)
$$\sum_{n=1}^{\infty} \frac{\pi e^n}{3^n}$$

(c)
$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2 + n + 1}}$$

(d)
$$\sum_{n=1}^{\infty} \ln \left(\frac{n+1}{n} \right)$$