Math 132 - Quiz 4

Name key

September 14, 2022

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due September 19.

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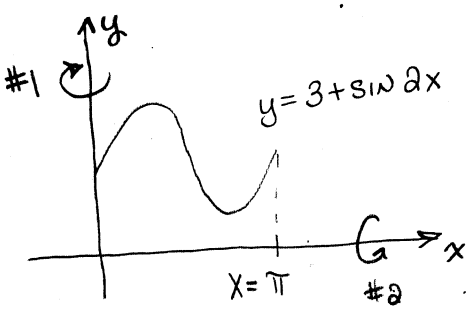
1. (2 points) The graph of $y = 3 + \sin(2x)$, $0 \leq x \leq \pi$, is rotated about the y-axis. Set up the definite integral that gives the area of the surface that is generated. Use technology to approximate the value of your integral.



$$\frac{dy}{dx} = 2\cos 2x$$

$$\text{SURFACE AREA} = 2\pi \int_0^{\pi} x\sqrt{1+4\cos^2 2x}\, dx$$

$$\approx 52.016 \text{ UNIT}^2$$

2. (2 points) Repeat the problem above, but instead rotate the graph about the x-axis.

$$\text{SURFACE AREA} = 2\pi \int_0^{\pi} (3+\sin 2x)\sqrt{1+4\cos^2 2x}\, dx$$

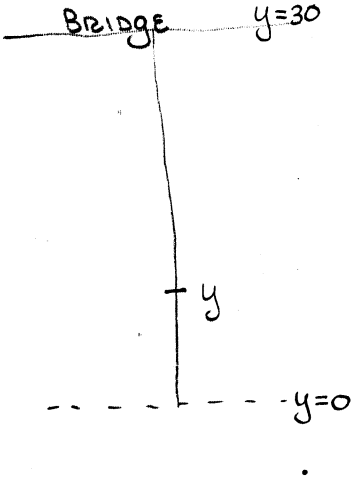
$$\approx 99.344 \text{ UNIT}^2$$

Turn over.

$9.75 \times 30 = 292.5$

3. (3 points) A chain weighs 9.75 lb/ft. It is hanging over the side of a bridge to a length of 30 ft. Determine the work required to pull up 20 ft of the chain.

LEAKY BUCKET APPROACH...



0 FT IN, 292.5 lb HANGING

30 FT IN, 0 lb HANGING

$\Downarrow$

WEIGHT $= 292.5 - 9.75y$

$$\text{WORK} = \int_0^{20} (292.5 - 9.75y)\, dy$$

$$= 292.5y - \frac{9.75}{2}y^2 \Big|_0^{20} = \boxed{3900 \text{ FT LB}}$$

2ND APPROACH -- MOVE TOP 30 FT AND BOTTOM 10 FT SEPARATELY

$$\int_{10}^{30} 9.75(30-y)\, dy + (9.75)(10)(20)$$

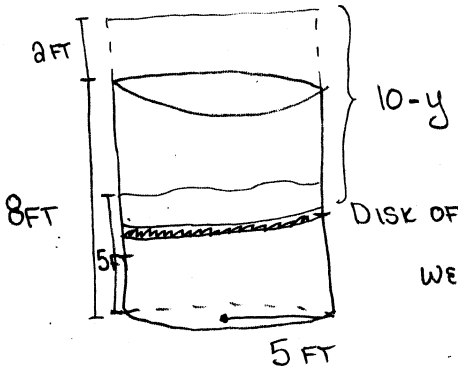
$$= \ldots = 3900 \text{ FT LB}$$

3RD APPROACH -- MOVE BOTTOM 20 FT TO TOP AND LEAVE TOP 10 FT ALONE

$$\int_0^{20} 9.75(30-y)\, dy = \ldots$$

$$= 3900 \text{ FT LB}$$

4. (3 points) A water tank has the shape of a right circular cylinder with diameter 10 ft and height 8 ft. Water weighing 62.4 pounds per cubic foot is stored in the tank to a depth of 5 ft. The water is pumped out to a platform that is 2 ft above the top of the tank. Find the amount of work required.



WEIGHS $62.4\pi(5)^2 dy$

$$\text{WORK} = \int_0^5 62.4\pi(5)^2\, dy\,(10-y)$$

$$= 1560\pi \int_0^5 (10-y)\, dy = 1560\pi \left(10y - \tfrac{1}{2}y^2\right)\Big|_0^5$$

$$= 1560\pi\left(50 - \frac{25}{2}\right) = \boxed{58{,}500\pi \text{ FT·LB} \approx 183{,}783 \text{ FT·LB}}$$