Math 131 - Final Exam

Spring 2026

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Use algebraic techniques (not a graph, table, or L'Hôpital's rule) to determine each limit.

(a) $\lim_{x\to 1} \dfrac{x^2+x-2}{2x^2-2}$

$\overset{0/0}{=} \lim_{x\to 1} \dfrac{(x+2)(x-1)}{2(x-1)(x+1)} = \boxed{\dfrac{3}{4}}$

(b) $\lim_{x\to 3} \dfrac{\sqrt{x+1}-2}{x-3}$

$\overset{0/0}{=} \lim_{x\to 3} \dfrac{x+1-4}{(x-3)(\sqrt{x+1}+2)} \cdot \dfrac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$

$= \dfrac{1}{\sqrt{4}+2} = \boxed{\dfrac{1}{4}}$

2. (10 points) Each function below has a single discontinuity. For each function, find the point of discontinuity and state the kind of discontinuity (removable, jump, or infinite). For full credit, you must show work.

(a) $g(x) = \dfrac{3x^2+x}{|x|} = \dfrac{x(3x+1)}{|x|}$

$\lim_{x\to 0^+} g(x) = 1$

$\lim_{x\to 0^-} g(x) = -1$

$x = 0$ Jump discont

(b) $h(x) = \dfrac{\sin(x-3)}{x-2}$

$\dfrac{\sin(-1) \neq 0}{0}$ Form

$x = 2$ Inf discont

(c) $f(x) = \begin{cases} x^2+1, & x<5 \\ 4x+6, & x>5 \end{cases}$

$\lim_{x\to 5^-} f(x) = 5^2+1 = 26$

$\lim_{x\to 5^+} f(x) = 4(5)+6 = 26$

$x = 5$ Removable

3. (10 points) An object is launched vertically upward from over the edge of a cliff. The object's height (in feet) after t seconds is given by

$$s(t) = -16t^2 + 64t + 192.$$

(a) What is the object's the maximum height? Give units with your answer.

$s'(t) = -32t + 64$

$s'(t) = 0 \Rightarrow t = 2$

$s(2) = 256$ FT

(b) What is object's velocity when it hits the ground? Give units with your answer.

$s(t) = 0 \Rightarrow -16(t^2 - 4t - 12) = -16(t-6)(t+2) = 0$

$t = 6, \quad t = -2$

$s'(6) = -32(6) + 64 = -128$ FT/sec

4. (10 points) Find the slope of the graph of the equation $x^3y^3 - y = x - 9$ at the point $(2, -1)$.

$$\frac{d}{dx}(x^3y^3 - y) = \frac{d}{dx}(x-9)$$

$$3x^2y^3 + 3x^3y^2\frac{dy}{dx} - \frac{dy}{dx} = 1$$

$$\frac{dy}{dx}(3x^3y^2 - 1) = 1 - 3x^2y^3$$

$$\frac{dy}{dx} = \frac{1 - 3x^2y^3}{3x^3y^2 - 1}.$$

$$\left.\frac{dy}{dx}\right|_{(x,y)=(2,-1)} = \frac{13}{23}$$

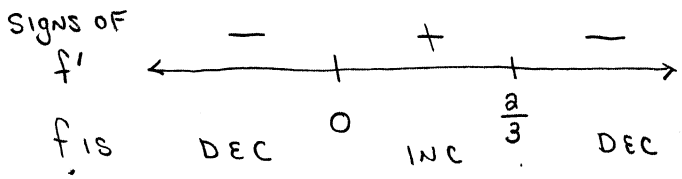
5. (10 points) Let $f(x) = 7 + 2x^2 - 2x^3$.

(a) Use the 1st derivative test to determine all relative extreme values.

$f'(x) = 4x - 6x^2$

$= -2x(3x-2) = 0$

$x = 0, \quad x = \frac{2}{3}$



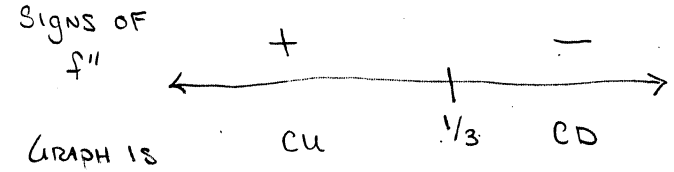
$f(0) = 7$ IS A REL MIN

$f\left(\frac{2}{3}\right) = \frac{197}{27} = 7.\overline{296}$ IS A REL. MAX.

(b) Use the 2nd derivative test to determine open intervals on which the graph of f is concave up/down.

$f''(x) = 4 - 12x$

$f''(x) = 0 \Rightarrow x = \frac{4}{12} = \frac{1}{3}$



GRAPH IS CU ON $(-\infty, 1/3)$ AND CD ON $(1/3, \infty)$.

6. (10 points) Use any analytical method (not a table or graph) to determine each limit.

(a) $\lim_{x\to 0^+} \frac{e^x - (1+x)}{x^3} \overset{0/0}{=} \lim_{x\to 0^+} \frac{e^x - 1}{3x^2} \overset{0/0}{=} \lim_{x\to 0^+} \frac{e^x}{6x} \overset{1/0^+}{=} +\infty$

L'HÔPITAL L'HÔPITAL

(b) $\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right) \overset{\infty\cdot 0}{=} \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = 1$

SAME LIMIT IS $\lim_{x\to 0^+} \frac{\sin x}{x} = 1$.

7. (10 points) Let $f(x) = \dfrac{1}{x}, \quad 1 \le x \le 2.$

(a) Use 5 subintervals of equal length and subinterval midpoints to compute the corresponding Riemann sum for f over $[1,2]$.

$\Delta x = \frac{1}{5} = 0.2$

PARTITION:

$1 < 1.2 < 1.4 < 1.6 < 1.8 < 2$

$c_1 = 1.1 \quad c_3 = 1.5 \quad c_5 = 1.9$

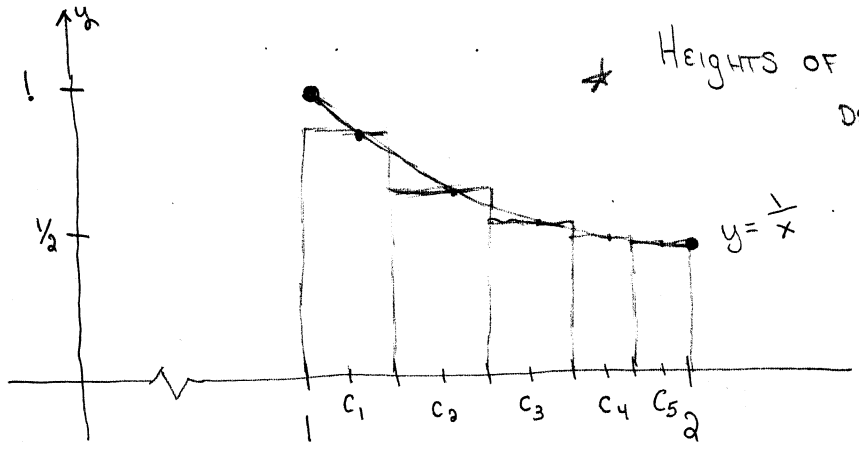
$c_2 = 1.3 \quad c_4 = 1.7$

RIEMANN SUM =

$0.2\left(\frac{1}{1.1} + \frac{1}{1.3} + \frac{1}{1.5} + \frac{1}{1.7} + \frac{1}{1.9}\right)$

$\approx \boxed{0.69191}$

(b) Sketch the graph of f over the interval $[1,2]$ and then sketch (in detail) the rectangles associated with your Riemann sum.



* HEIGHTS OF RECTANGLES DETERMINED AT MIDPOINTS. *

(c) Use the Fundamental Theorem of Calculus to briefly explain why your answer in part (a) should be close to $\ln 2$.

$$\int_1^2 \frac{1}{x}\,dx = \ln x \Big|_1^2 = \ln 2$$

8. (10 points) Evaluate each indefinite integral. (Be sure to check your answer by differentiation.)

(a) $\int \left(6x^3 + \sqrt{x} + \frac{5}{x^2}\right) dx$ $= \int \left(6x^3 + x^{1/2} + 5x^{-2}\right) dx$

$= \frac{6}{4}x^4 + \frac{x^{3/2}}{3/2} + \frac{5x^{-1}}{-1} + C$

$= \boxed{\frac{3}{2}x^4 + \frac{2}{3}x^{3/2} - \frac{5}{x} + C}$

(b) $\int (e^{4x} + \sec^2 x)\, dx$

$= \boxed{\frac{1}{4}e^{4x} + \tan x + C}$

9. (10 points) Evaluate each indefinite integral by using the given substitution.

(a) $\int x\cos(2x^2)\, dx$, use $u = 2x^2$

$du = 4x\, dx$

$\frac{1}{4}du = x\, dx$

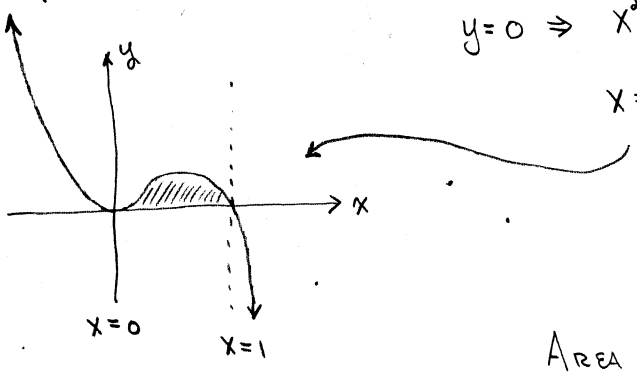
$\frac{1}{4}\int \cos u\, du = \frac{1}{4}\sin u + C = \boxed{\frac{1}{4}\sin(2x^2) + C}$

(b) $\int e^{\sin x}\cos x\, dx$, use $u = \sin x$

$du = \cos x\, dx$

$\int e^u\, du = e^u + C = \boxed{e^{\sin x} + C}$

10. (10 points) Sketch the graph of $y = x^2 - x^3$ carefully showing the bounded region in the first quadrant between the graph and the x-axis. Then use a definite integral to find the area of that bounded region.



$y=0 \Rightarrow x^2(1-x)=0$

$x=0,\ x=1$

$$\text{AREA} = \int_0^1 (x^2 - x^3)\,dx$$

$$= \frac{1}{3}x^3 - \frac{1}{4}x^4 \Big|_0^1$$

$$= \left(\frac{1}{3} - \frac{1}{4}\right) - (0-0)$$

$$= \boxed{\frac{1}{12}}$$