Math 131 - Test 1

February 11, 2026

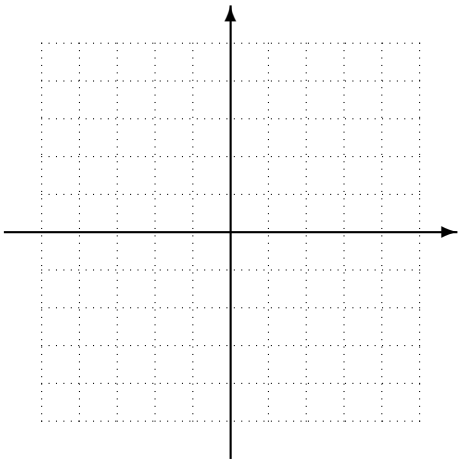
Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

---

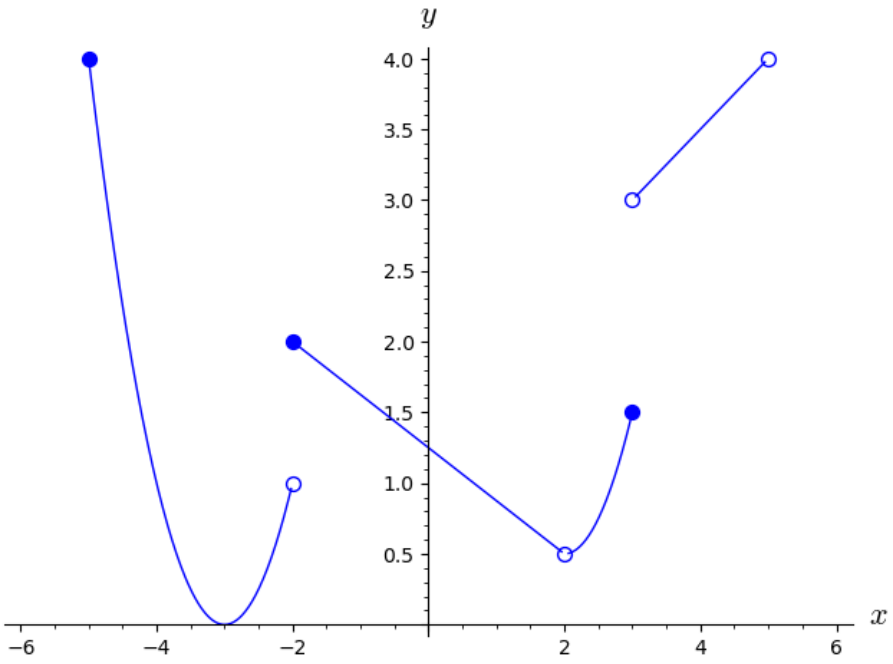
1. (5 points) Describe a reason that a limit may fail to exit at a point. Then sketch the graph of a function that illustrates your reason.



2. (8 points) Use a table of numerical values to approximate the following limit. Your table must show function values at six or more points.

$$\lim_{x \to 0} \frac{1 - \cos x}{x \sin x}$$

3. (12 points) The function f is defined on the interval $[-5, 5)$. Its graph is shown below. Estimate each of the following or explain why it does not exist.



(a) $\lim_{x \to -3} f(x)$

(b) $\lim_{x \to 2} f(x)$

(c) $\lim_{x \to 4} f(x)$

(d) $\lim_{x \to -5} f(x)$

(e) $\lim_{x \to -2^-} f(x)$

(f) $\lim_{x \to -2^+} f(x)$

4. (6 points) These limits DO NOT EXIST. Carefully explain why each limit fails to exist.

   (a) $\lim_{x\to 3} \dfrac{\sqrt{x-3}}{x}$

   (b) $\lim_{x\to 0} \cos\left(\dfrac{1}{x^2}\right)$

5. (9 points) Given the following information,

$$f(2) = 8, \qquad \lim_{x\to 2} f(x) = 5, \qquad g(2) = -3, \qquad \lim_{x\to 2} g(x) = 10,$$

   find the value of each expression below. To receive credit, you must show how you used the limit laws.

   (a) $\lim_{x\to 2} 3f(x)g(x)$

   (b) $\lim_{x\to 2} \dfrac{x^2+1}{g(x)}$

   (c) $\lim_{x\to 2} \left(f(x) - g(x)\right)^2$

6. (5 points) Determine whether each statement is true (T) or false (F).

   (a) _____ The limit of a polynomial function can always be found by direct substitution.

   (b) _____ If f is defined at $x = 2$, then $\lim_{x\to 2} f(x)$ must exist.

   (c) _____ $\lim_{x\to 0} \sqrt[3]{x} = 0$

   (d) _____ If $\lim_{x\to 3} g(x) = 7$, then $\lim_{x\to 3^+} g(x) = 7$.

   (e) _____ The limit of any one of the six basic trigonometric functions can always be found by direct substitution.

7. (24 points) **Determine each limit analytically**, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE. You will not be given credit if you get your answer from a table of values or a graph.

(a) $\lim_{x \to -2} \dfrac{(x-1)^2 + 6x + 3}{x^2 + 3x + 2}$

(b) $\lim_{y \to 13} \dfrac{\sqrt{y+3} - 4}{y - 13}$

(c) $\lim_{t \to 4^-} \dfrac{t^2 - 16}{|t - 4|}$

(d) $\lim_{x \to 0} \dfrac{\tan x \, \cos x}{5x}$

8. (4 points) It is not difficult to show that $f(x) = x^2 \sin \dfrac{1}{x}$ satisfies the following inequalities for all nonzero x-values.

$$-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$$

Determine $\lim\limits_{x \to 0} f(x)$ and explain your reasoning.

9. (12 points) For each part of this problem, **determine analytically** whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

(a) $\lim\limits_{x \to 4^+} \dfrac{x^2}{4 - x}$

(b) $\lim\limits_{x \to 1} \dfrac{2}{x^2 - 1}$

(c) $\lim\limits_{x \to \pi^-} \dfrac{x}{\sin x}$

(d) $\lim\limits_{x \to -3} \dfrac{|x + 7|}{|x + 3|}$

10. (4 points) Determine all vertical asymptotes of the graph of $R(x) = \dfrac{x^2 + 6x + 8}{x^2 - 4}$. Show work to support your answer.

11. (4 points) Use the definition of continuity to show that $g(x) = \dfrac{x^2 + 3x}{x}$ is continuous at $x = 2$.

12. (7 points) Find the number k so that f is continuous at $x = 5$. For full credit, your work must show how you are using limits and the definition of continuity.

$$f(x) = \begin{cases} x^2 - 3x + 5, & x \leq 5 \\ x^3 + kx, & x > 5 \end{cases}$$