Math 131 - Test 1

February 11, 2026

Name key

Score

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

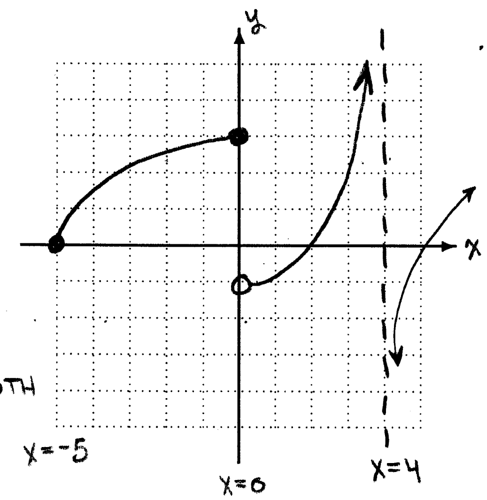
1. (5 points) Describe a reason that a limit may fail to exit at a point. Then sketch the graph of a function that illustrates your reason.

HERE ARE THREE OF OUR FOUR REASONS

① $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

② $\lim_{x\to 4} f(x)$ DNE BECAUSE OF UNBOUNDED GROWTH AS $x \to 4$

④ $\lim_{x\to -5} f(x)$ DNE BECAUSE f IS NOT DEFINED FOR $x < -5$



2. (8 points) Use a table of numerical values to approximate the following limit. Your table must show function values at six or more points.

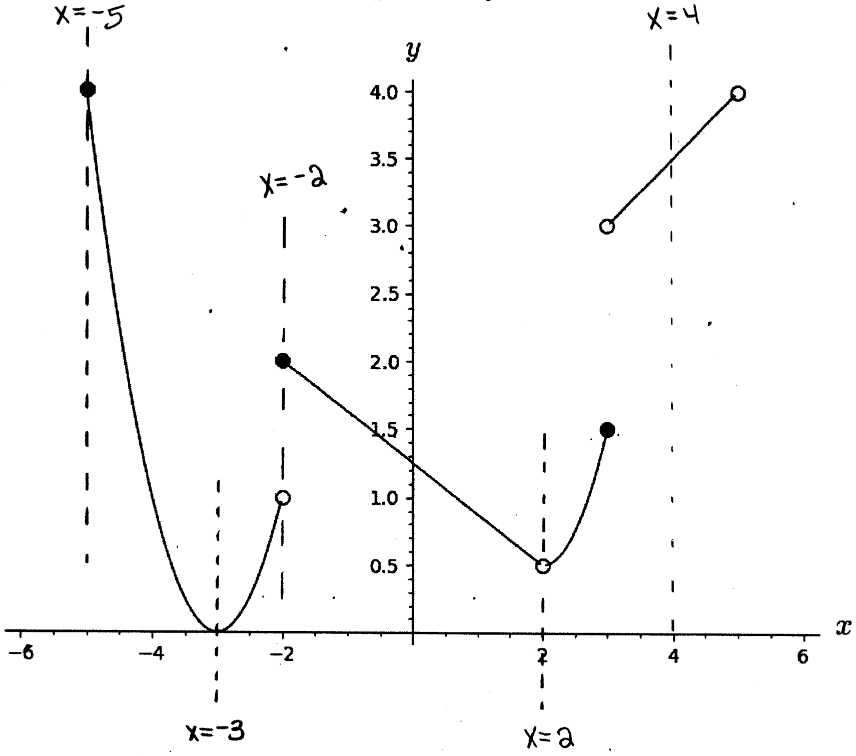
$$\lim_{x\to 0} \frac{1-\cos x}{x \sin x}$$

x	$\frac{1-\cos x}{x \sin x}$
0.1	0.500417
0.01	0.500004
0.001	0.500000
-0.1	0.500417
-0.01	0.500004
-0.001	0.500000

IT LOOKS VERY MUCH LIKE

$$\lim_{x\to 0} \frac{1-\cos x}{x \sin x} = \frac{1}{2}$$

3. (12 points) The function f is defined on the interval $[-5, 5)$. Its graph is shown below. Estimate each of the following or explain why it does not exist.



(a) $\lim_{x \to -3} f(x)$ $= \boxed{0}$

(b) $\lim_{x \to 2} f(x)$ $= \boxed{0.5}$

(c) $\lim_{x \to 4} f(x)$ $\approx \boxed{3.5}$

(d) $\lim_{x \to -5} f(x)$ $\boxed{\text{DNE}}$ f IS NOT DEFINED TO THE LEFT OF X=-5 ⇒ CANNOT HAVE A LIMIT AT X=-5

(e) $\lim_{x \to -2^-} f(x)$ $= \boxed{1}$

(f) $\lim_{x \to -2^+} f(x)$ $= \boxed{2}$

4. (6 points) These limits DO NOT EXIST. Carefully explain why each limit fails to exist.

(a) $\lim_{x\to 3} \frac{\sqrt{x-3}}{x}$

FAILURE #4

$\sqrt{X-3}$ IS NOT DEFINED WHEN $X < 3$.
IN THIS CASE, $\lim_{X\to 3^+} \frac{\sqrt{X-3}}{X} = 0$, BUT $\lim_{X\to 3^-} \frac{\sqrt{X-3}}{X}$ MAKES NO SENSE.

(b) $\lim_{x\to 0} \cos\left(\frac{1}{x^2}\right)$

FAILURE #3

AS $X \to 0$ $\frac{1}{X^2}$ GROWS WITHOUT BOUND, BUT WHILE THAT HAPPENS, THE VALUES OF $\cos\left(\frac{1}{X^2}\right)$ OSCILLATE WITHOUT APPROACHING AN FIXED VALUE.

5. (9 points) Given the following information,

$$f(2) = 8, \qquad \lim_{x\to 2} f(x) = 5, \qquad g(2) = -3, \qquad \lim_{x\to 2} g(x) = 10,$$

find the value of each expression below. To receive credit, you must show how you used the limit laws.

(a) $\lim_{x\to 2} 3f(x)g(x) = 3 \lim_{X\to 2} f(x) \lim_{X\to 2} g(x) = 3(5)(10) = \boxed{150}$

(b) $\lim_{x\to 2} \frac{x^2+1}{g(x)} = \frac{\left(\lim_{X\to 2} X\right)^2 + \lim_{X\to 2} 1}{\lim_{X\to 2} g(x)} = \frac{(2)^2 + 1}{10} = \frac{5}{10} = \boxed{\frac{1}{2}}$

(c) $\lim_{x\to 2} (f(x) - g(x))^2$

$= \left[\lim_{X\to 2} f(x) - \lim_{X\to 2} g(x)\right]^2 = (5-10)^2 = \boxed{25}$

6. (5 points) Determine whether each statement is true (T) or false (F).

(a) __T__ The limit of a polynomial function can always be found by direct substitution.

(b) __F__ If f is defined at $x = 2$, then $\lim_{x\to 2} f(x)$ must exist.

(c) __T__ $\lim_{x\to 0} \sqrt[3]{x} = 0$

(d) __T__ If $\lim_{x\to 3} g(x) = 7$, then $\lim_{x\to 3^+} g(x) = 7$.

(e) __F__ The limit of any one of the six basic trigonometric functions can always be found by direct substitution.

7. (24 points) **Determine each limit analytically**, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE. You will not be given credit if you get your answer from a table of values or a graph.

(a) $\lim_{x\to-2} \frac{(x-1)^2+6x+3}{x^2+3x+2}$ $\quad 0/0$

$$= \lim_{x\to-2} \frac{x^2-2x+1+6x+3}{x^2+3x+2} = \lim_{x\to-2} \frac{x^2+4x+4}{x^2+3x+2}$$

$$= \lim_{x\to-2} \frac{(x+2)\cancel{(x+2)}}{(x+1)\cancel{(x+2)}} = \boxed{0}$$

(b) $\lim_{y\to13} \frac{\sqrt{y+3}-4}{y-13} \cdot \frac{\sqrt{y+3}+4}{\sqrt{y+3}+4}$ $\quad 0/0$ $= \lim_{y\to13} \frac{y+3-16}{(y-13)(\sqrt{y+3}+4)}$

$$= \lim_{y\to13} \frac{\cancel{y-13}}{\cancel{(y-13)}(\sqrt{y+3}+4)} = \boxed{\frac{1}{8}}$$

(c) $\lim_{t\to4^-} \frac{t^2-16}{|t-4|}$ $\quad 0/0$ $= \lim_{t\to4^+} \frac{\cancel{(t-4)}(t+4)}{-\cancel{(t-4)}} = \boxed{-8}$

LEFT OF $t=4$,

$|t-4| = -(t-4)$

(d) $\lim_{x\to0} \frac{\tan x \cos x}{5x}$ $\quad 0/0$ $= \lim_{x\to0} \left(\frac{\frac{\sin x}{\cancel{\cos x}} \cancel{\cos x}}{5x} \right) = \lim_{x\to0} \frac{\sin x}{5x}$

$$= \frac{1}{5} \lim_{x\to0} \frac{\sin x}{x}$$

$$= \frac{1}{5}(1) = \boxed{\frac{1}{5}}$$

8. (4 points) It is not difficult to show that $f(x) = x^2 \sin \frac{1}{x}$ satisfies the following inequalities for all nonzero x-values.

$$-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$$

Determine $\lim_{x \to 0} f(x)$ and explain your reasoning.

SINCE

$\lim_{x \to 0} (-x^2) = 0$ AND $\lim_{x \to 0} x^2 = 0$, $\boxed{\lim_{x \to 0} x^2 \sin \frac{1}{x} = 0 \text{ BY SQUEEZE THM}}$

9. (12 points) For each part of this problem, **determine analytically** whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

(a) $\lim_{x \to 4^+} \frac{x^2}{4-x}$ 16/0 UNBOUNDED GROWTH

TO THE RIGHT OF X=4:

$\frac{x^2}{4-x} = \frac{\text{POS}}{\text{NEG}} = \text{NEG}$

$\boxed{\lim_{x \to 4^+} \frac{x^2}{4-x} = -\infty}$

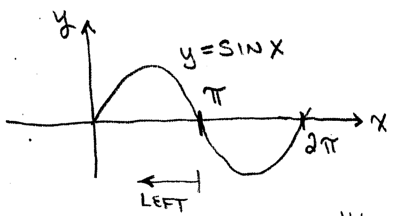
(b) $\lim_{x \to 1} \frac{2}{x^2 - 1}$ 2/0

LEFT OF X=1: $\frac{2}{x^2-1} = \frac{\text{POS}}{\text{NEG}} \Rightarrow \lim_{x \to 1^-} \frac{2}{x^2-1} = -\infty$

RIGHT OF X=1: $\frac{2}{x^2-1} = \frac{\text{POS}}{\text{POS}} \Rightarrow \lim_{x \to 1^+} \frac{2}{x^2-1} = +\infty$

LIMIT AT X=1: DNE

(c) $\lim_{x \to \pi^-} \frac{x}{\sin x}$ π/0



LEFT OF X=π: $\frac{x}{\sin x} = \frac{\text{POS}}{\text{POS}} = \text{POS}$

$\boxed{\lim_{x \to \pi^-} \frac{x}{\sin x} = +\infty}$

(d) $\lim_{x \to -3} \frac{|x+7|}{|x+3|}$ 4/0

$|x+7|$ AND $|x+3|$ ARE BOTH ALWAYS POSITIVE ON BOTH SIDES OF $x = -3$...

$\boxed{\lim_{x \to -3} \frac{|x+7|}{|x+3|} = +\infty}$

10. (4 points) Determine all vertical asymptotes of the graph of $R(x) = \dfrac{x^2+6x+8}{x^2-4}$. Show work to support your answer.

$$R(x) = \frac{\cancel{(x+2)}(x+4)}{\cancel{(x+2)}(x-2)}$$

THERE IS A LIMIT AT $X = -2$

UNBOUNDED GROWTH AT $X=2$

$\boxed{X = 2 \text{ IS THE ONLY V.A.}}$

11. (4 points) Use the definition of continuity to show that $g(x) = \dfrac{x^2+3x}{x}$ is continuous at $x = 2$.

$$\underbrace{\lim_{x\to 2} g(x) = \lim_{x\to 2} \frac{x^2+3x}{x} = \frac{4+6}{2} = 5}_{} = g(2) \checkmark$$

CONTINUOUS AT $X=2$

12. (7 points) Find the number k so that f is continuous at $x = 5$. For full credit, your work must show how you are using limits and the definition of continuity.

$$f(x) = \begin{cases} x^2 - 3x + 5, & x \le 5 \\ x^3 + kx, & x > 5 \end{cases}$$

$$f(5) = (5)^2 - 3(5) + 5 = 15$$

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-} (x^2 - 3x + 5) = 15$$

MUST BE EQUAL.

$$\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+} (x^3 + kx) = (5)^3 + 5k = 125 + 5k$$

$$125 + 5k = 15$$

$$5k = -120$$

$$\boxed{k = -22}$$