Math 131 - Quiz 4

February 18, 2026

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (6 points) Find and classify (removable, infinite, or jump) each discontinuity. Your work must show how you are using limits.

THE ONLY POSSIBLE DISCONTS ARE AT $x = 1, 2, 4$. LET'S CHECK THEM.

$$f(x) = \begin{cases} x^2 + 2x + 3, & x < 1 \\ x^3 + 5, & 1 \le x < 2 \\ 6x + \cos(\pi x), & 2 < x \le 4 \\ x + 8\sqrt{x}, & x > 4 \end{cases}$$

$f(1) = (1)^3 + 5 = 6$

$\lim_{x \to 1^-} f(x) = (1)^2 + 2(1) + 3 = 6$

$\lim_{x \to 1^+} f(x) = (1)^3 + 5 = 6$

f IS CONTINUOUS AT $x = 1$

$f(2)$ DNE

$\lim_{x \to 2^-} f(x) = (2)^3 + 5 = 13$

$\lim_{x \to 2^+} f(x) = 6(2) + \cos(2\pi) = 13$

LIMIT EXISTS. $f(2)$ DOES NOT.

$x = 2$ IS A REMOVABLE DISCONT.

$f(4) = 6(4) + \cos(4\pi) = 25$

$\lim_{x \to 4^-} f(x) = 6(4) + \cos(4\pi) = 25$

$\lim_{x \to 4^+} f(x) = 4 + 8\sqrt{4} = 20$

$x = 4$ IS A JUMP DISCONT.

2. (2 points) Let $f(x) = x^5 - 6x + 3$. Use the intermediate value theorem to identify an interval, $[a, b]$, on which $f(x) = 0$ has a solution. Explain your reasoning.

f IS CONT. EVERYWHERE.

$f(1) = (1)^5 - 6(1) + 3 = -2$

$f(2) = (2)^5 - 6(2) + 3 = 23$

$f(1)$ IS NEG. $f(2)$ IS POS.

$f(x) = 0$ HAS A SOLUTION ON $[1, 2]$.

3. (2 point) Carefully state the limit definition of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \text{ PROVIDED THIS LIMIT EXISTS.}$$