Math 131 - Quiz 3

February 4, 2026

Name ~~Key~~ Key

Score ______

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Determine each limit analytically.

   (a) $\lim_{x\to 4} \frac{x(4-x)}{2-\sqrt{x}}$ $0/0$ $\cdot \frac{2+\sqrt{x}}{2+\sqrt{x}} = \lim_{x\to 4} \frac{x\cancel{(4-x)}(2+\sqrt{x})}{\cancel{4-x}} = 4(2+\sqrt{4}) = \boxed{16}$

   (b) $\lim_{x\to 0} \frac{\sin 5x}{x}$ $0/0$

   $= \lim_{x\to 0} \frac{5\sin 5x}{5x} = 5 \lim_{x\to 0} \frac{\sin 5x}{5x} = (5)(1) = \boxed{5}$

2. (4 points) Determine each limit below analytically or explain why it does not exist.

$$f(x) = \begin{cases} x^2 - 5x + 6, & x < 2 \\ x^3 - 1, & x > 2 \end{cases}$$

   (a) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x^3 - 1) = (2)^3 - 1 = 8 - 1 = \boxed{7}$

   (b) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x^2 - 5x + 6) = (2)^2 - 5(2) + 6 = 4 - 10 + 6 = \boxed{0}$

   (c) $\lim_{x\to 2} f(x)$ $\boxed{\text{DNE}}$ LIMIT FROM RIGHT $\neq$ LIMIT FROM LEFT (SEE (a) & (b).)

   (d) $\lim_{x\to 1} f(x) = \lim_{x\to 1} (x^2 - 5x + 6) = (1)^2 - 5(1) + 6 = \boxed{2}$

3. (1 point) Suppose that $\frac{1}{2} - \frac{x^2}{24} < \frac{1-\cos x}{x^2} < \frac{1}{2}$ for values of x close to $x = 0$.

   What can you say about the limit below? What is the name of the theorem you used?

$$\lim_{x\to 0} \frac{1-\cos x}{x^2}$$

SINCE $\lim_{x\to 0} \frac{1}{2} - \frac{x^2}{24} = \frac{1}{2}$

AND $\lim_{x\to 0} \frac{1}{2} = \frac{1}{2}$,

$\boxed{\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2} \text{ BY SQUEEZE THM}}$