

Math 131 - Test 3
April 16, 2025

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Let $f(x) = -x^3 - 3x + 7$. Find $(f^{-1})'(11)$.

2. (6 points) Let $g(x) = \frac{\cos^{-1} x}{\cos x}$. Find $g'(x)$. Then find the slope of the line tangent to the graph of g at the point where $x = 0$.

3. (5 points) Determine the derivative of $y = e^{x^{1/3}} + e^{x/3}$.

4. (6 points) Find an equation of the line tangent to the graph of $y = (e^x + 1)^2$ at the point where $x = 0$.

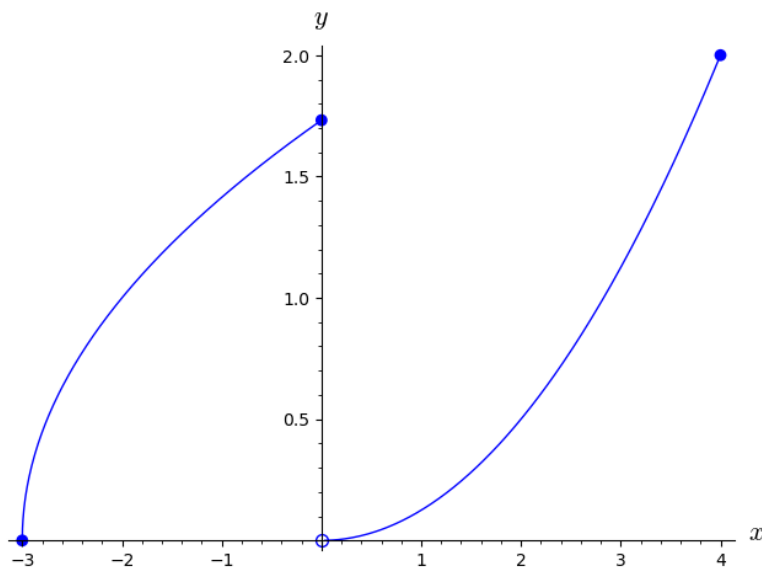
5. (6 points) Find the instantaneous rate of change of $g(x) = \ln(xe^{-x^3})$ at the point where $x = 1$.

6. (8 points) Use logarithmic differentiation to find dy/dx when $y = \frac{(x^2 + 1)^x}{x^2}$.
(Helpful hint: You'll need the product rule for the derivative of $x \ln(x^2 + 1)$.)

7. (3 points) Determine the derivative of $f(x) = \log_3 x$.
8. (7 points) Determine the linearization of $h(x) = \frac{1}{3+x}$ at $x = 0$. Then use your linearization to approximate $h(0.1)$ (which should be pretty close to $\frac{1}{3.1}$).
9. (6 points) Use a linearization to approximate $(7.9)^{2/3}$.
10. (5 points) The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$. Use differentials to approximate the change in volume as the radius decreases from 3.00 cm to 2.95 cm. Include units on your final answer.

11. (4 points) Let $f(x) = \sqrt{x}$. Find $f'(x)$ and then explain why $x = 0$ IS NOT a critical number of f .

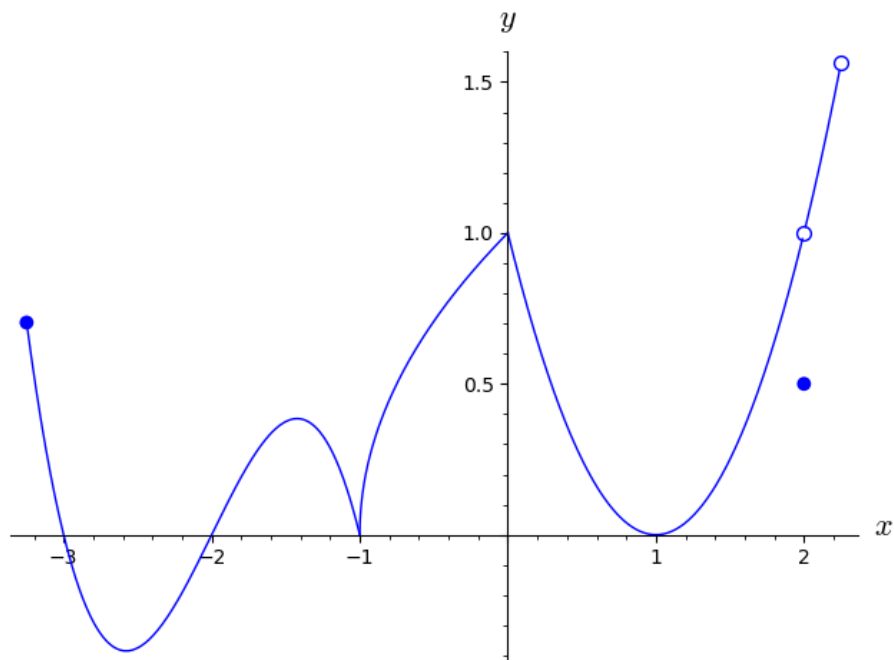
12. (4 points) The function f has domain $[-3, 4]$. The graph of f is shown below. Explain why the maximum value at $x = 4$ IS NOT a relative maximum, but the maximum value at $x = 0$ IS a relative maximum.



13. (8 points) Let $h(x) = x\sqrt{x+1}$. Find all numbers in the domain of h for which $h'(x) = 0$ or $h'(x)$ does not exist. Then determine all critical numbers of h .

14. (10 points) Use calculus techniques (showing all work) to determine the absolute maximum and minimum values of $f(x) = 3x^4 + 4x^3 - 12x^2 + 10$ on $[-3, 2]$.

15. (6 points) The domain of g is the interval $[-3.25, 2.25)$, and the graph of g is shown below. Use the graph to estimate the critical numbers of g .



16. (10 points) Let $f(x) = (x - 2)^2(x + 3)$. Use calculus techniques (showing all work) to determine open intervals on which f is increasing/decreasing. Also identify all relative extreme values.