Math 131 - Assignment 12

May 1, 2024

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. Use extra paper as necessary. This assignment is due May 8.

1. Let $F(x) = \int_x^{\pi/4} t \tan t \, dt$. Determine $F'(x)$.

$$= -\int_{\pi/4}^{x} t \tan t \, dt = \boxed{-x \tan x}$$

2. Consider the function $F(x) = \int_2^x (t + t^2 + t^3)\, dt$.

   (a) Evaluate $F(1)$.

$$\int_2^1 (t + t^2 + t^3)\, dt = \frac{1}{2}t^2 + \frac{1}{3}t^3 + \frac{1}{4}t^4 \Big|_2^1 = \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) - \left(\frac{4}{2} + \frac{8}{3} + \frac{16}{4}\right)$$

$$= -\frac{91}{12}$$

   (b) Find $F'(x)$ by evaluating the integral and then differentiating.

$$F(x) = \int_2^x (t + t^2 + t^3)\, dt = \frac{1}{2}t^2 + \frac{1}{3}t^3 + \frac{1}{4}t^4 \Big|_2^x = \left(\frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4\right) - \left(\frac{4}{2} + \frac{8}{3} + \frac{16}{4}\right)$$

$$= \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} - \frac{26}{3}$$

$$F'(x) = x + x^2 + x^3$$

   (c) Find $F'(x)$ by using the 2nd Fundamental Theorem of Calculus.

$$\frac{d}{dx}\int_2^x (t + t^2 + t^3)\, dt = \boxed{x + x^2 + x^3}$$

3. Use substitution to evaluate each definite integral.

(a) $\int_0^1 6x^2(x^3+12)^{118}\,dx$ $= 2\int_{12}^{13} u^{118}\,du = \frac{2}{119}u^{119}\Big|_{12}^{13}$

$u = x^3+12$

$du = 3x^2\,dx$

$2du = 6x^2\,dx$

$= \frac{2}{119}\left[13^{119} - 12^{119}\right]$

(b) $\int_1^2 3x\,e^{x^2}\,dx$ $= \frac{3}{2}\int_1^4 e^u\,du = \frac{3}{2}e^u\Big|_1^4$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{3}{2}\left[e^4 - e\right]$

4. Use substitution to evaluate each indefinite integral.

(a) $\int \frac{e^{1/x}}{x^2}\,dx$ $= -\int e^u\,du = -e^u + C$

$u = \frac{1}{x}$

$du = -\frac{1}{x^2}dx$

$-du = \frac{1}{x^2}dx$

$= -e^{1/x} + C$

(b) $\int \frac{x}{x^2+5}\,dx$ $= \frac{1}{2}\int \frac{1}{u}\,du$

$u = x^2+5$

$= \frac{1}{2}\ln|u| + C$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\ln(x^2+5) + C$