Math 131 - Test 1

February 20, 2020

Name ________________________________

Score __________

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

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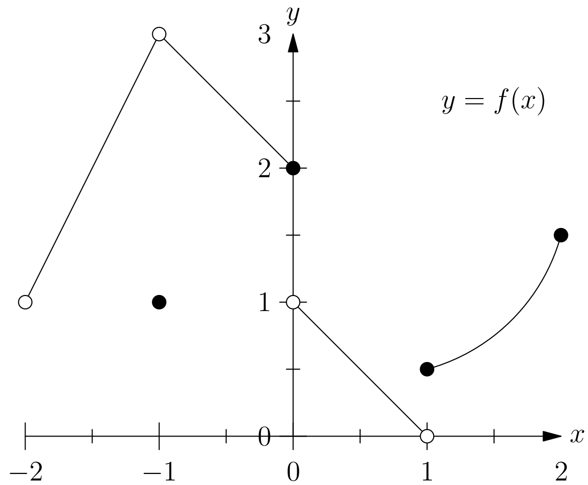
1. (10 points) Referring to the graph shown below, determine each of the following or explain why it does not exist.



(a) $\lim_{x \to -1} f(x)$

(b) $\lim_{x \to 0^-} f(x)$

(c) $f(1)$

(d) $\lim_{x \to 0^+} f(x)$

(e) $\lim_{x \to -2} f(x)$

2. (9 points) Suppose that $\lim_{x \to 2} f(x) = 3$ and $\lim_{x \to 2} g(x)$ exists. Determine each limit.

   (a) $\lim_{x \to 2} (x^2 f(x) + g(x) \sin \pi x)$

   (b) $\lim_{x \to 2} g(x)$ if $\lim_{x \to 2} \dfrac{1}{(g(x))^2} = 5$

   (c) $\lim_{x \to 2} g(x)$ if $\lim_{x \to 2} \dfrac{f(x)}{g(x)}$ does not exist

3. (6 points) Use a table of numerical values to approximate the following limit. Your table must show function values at four or more points.

$$\lim_{x \to 0^+} \frac{5^x - 1}{3x}$$

4. (6 points) Determine all points at which g is discontinuous. Carefully explain your reasoning.

$$g(x) = \begin{cases} x^2 + 2, & x \leq 0 \\ 1 + \cos x, & 0 < x < \pi \\ 1 + x \sin x, & x \geq \pi \end{cases}$$

5. (24 points) Determine each limit analytically, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE.

(a) $\displaystyle\lim_{x\to 0} \frac{(x-2)^2-4}{x}$

(b) $\displaystyle\lim_{k\to 4} \frac{\sqrt{k}-2}{k-4}$

(c) $\displaystyle\lim_{x\to 4^-} \left(\frac{x-4}{x^2-8x+16}\right)$

(d) $\displaystyle\lim_{z\to 6} \frac{(z-4)^2+2(z+1)}{z+3}$

6. (4 points) Given that $-x^2 \le x^2 \cos\frac{1}{x} \le x^2$ when $x \neq 0$, compute $\lim_{x\to 0} x^2 \cos\frac{1}{x}$. Explain. (Can you state the name of the theorem you used?)

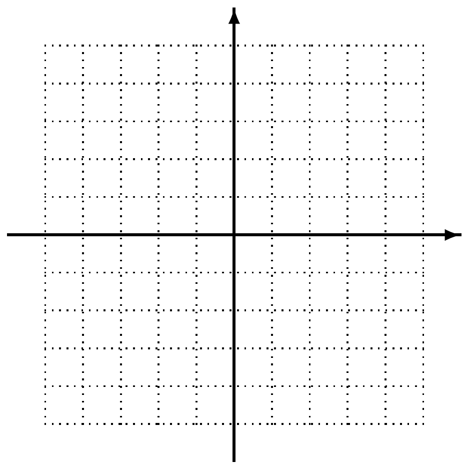
7. (5 points) Use the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$ to compute $\lim_{x\to 0} \left(\frac{3\tan 2x}{x}\right)$.

8. (5 points) Each row of the table below gives some information about a function f. Fill in each blank entry with an appropriate word or number. In some cases there may be more than one correct answer.

Continuous at $x = 2$	$f(2)$	$\lim_{x\to 2^-} f(x)$	$\lim_{x\to 2^+} f(x)$
Yes	5	5	
No	7		7
No		-1	-1
	2	2	2
Yes		1	

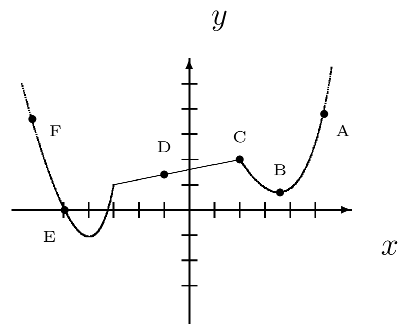
9. (6 points) Sketch the graph of a function f such that

- f is defined for all real numbers between -5 and 5,
- $f(-2) = 3$,
- f has a removable discontinuity at $x = -2$,
- $\lim_{x \to 3^-} f(x) = \infty$, and
- $\lim_{x \to 3^+} f(x) = -1$.



10. (6 points) Consider the rational function $R(x) = \dfrac{2x+4}{x^2+3x+2}$. Find all points at which R is discontinuous, and state whether each discontinuity is removable or non-removable.

11. (6 points) Consider the function f whose graph is shown below.



Referring to the labeled points, find a point at which

(a) $f'(x) = 0$

(b) $0 < f'(x) < 1$

(c) $f'(x) > 1$

(d) $f(x) = 0$

(e) $f'(x) < 0$

(f) $f'(x)$ is not defined

Formal Definition of Limit: The statement $\lim_{x \to c} f(x) = L$ means that for each $\epsilon > 0$, there exists a $\delta > 0$ such that $0 < |x - c| < \delta$ implies that $|f(x) - L| < \epsilon$.

12. (5 points) Compute $\lim_{x \to -2} (2x - 5)$ and then, referring to the formal definition of limit, find a δ that corresponds to an arbitrary positive ϵ.

13. (8 points) Let $f(x) = x^2 - 2x$. Use the limit definition of derivative to determine $f'(x)$.