Math 131 - Quiz 7

April 30, 2020

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. You must work individually on this quiz. This quiz is due no later than May 5.

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1. (2 points) Let $f(x) = 6x^2 - \sec x \tan x$. Determine the antiderivative of f whose graph passes through the point $(0, 5)$.

$$F(x) = \int (6x^2 - \sec x \tan x)\, dx = 2x^3 - \sec x + C$$

$$F(0) = 5 \Rightarrow 2(0)^3 - \sec 0 + C = 5$$

$$0 - 1 + C = 5$$

$$C = 6$$

$$\boxed{F(x) = 2x^3 - \sec x + 6}$$

2. (2 points) Suppose f and g are functions that satisfy $\frac{d}{dx}g(x) = -2f(x)$.

   Evaluate $\int 5f(x)\, dx$. (Your answer should be written in terms of the function g.)

$$\int -2f(x)\, dx = g(x) + C$$

$$\text{So,} \quad -\frac{5}{2}\underbrace{\int -2f(x)\, dx}_{g(x)} = \int 5f(x)\, dx = \boxed{-\frac{5}{2}g(x) + C}$$

3. (2 points) Use differentiation to check whether the following statement is true or false. Show your work.

$$\int \ln x\, dx = x \ln x + x + C$$

$$\frac{d}{dx}(x \ln x + x + C)$$

$$= (1)\ln x + x\left(\frac{1}{x}\right) + 1 + 0$$

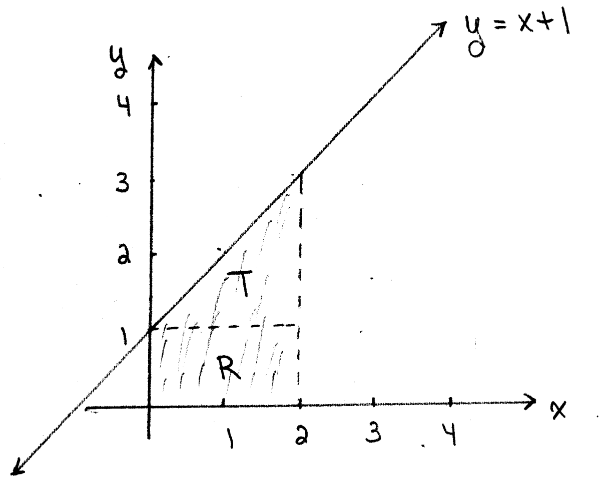
$$= \ln x + 2 \neq \ln x$$

STATEMENT IS FALSE.

4. (2 points) Let $f(x) = -x^2 + 3x - 2$. Use 4 rectangles (of equal base length) to estimate the area of the region between the graph of $y = f(x)$ and $y = 0$. For the heights of your rectangles, use function values at the left endpoints of the subintervals. (Using the notation of our textbook, you are computing L_4.) Draw the corresponding picture.

SEE ATTACHMENT.

5. (2 points) Sketch the graph of $y = x + 1$ over the interval $[0, 2]$. Then use the area concept to compute $\int_0^2 (x+1)\,dx$. (Your work must show how you used area to get your answer.)



$$\int_0^2 (x+1)\,dx = \text{AREA OF REGION}$$

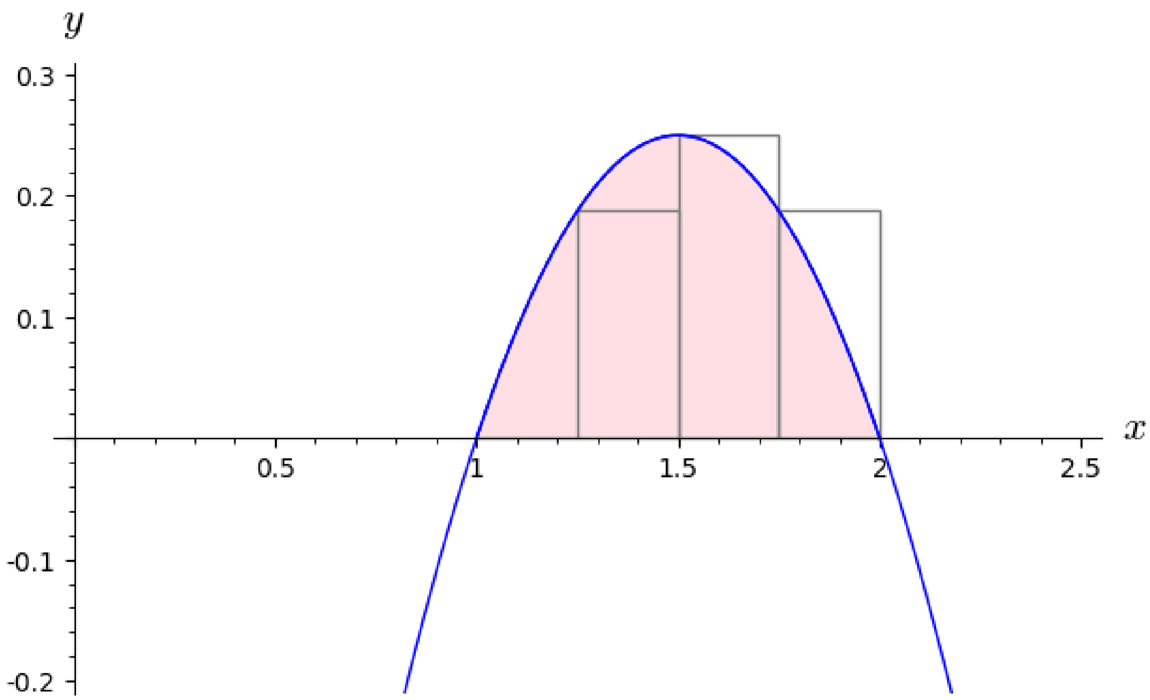
$$= T + R$$

$$= \frac{1}{2}(2)(2) + (2)(1)$$

$$= \boxed{4}$$

Problem 4

The graph of $f(x) = -x^2 + 3x - 2$ looks like this:



The region between the graph and the x-axis extends from $x = 1$ to $x = 2$. So we use 4 subintervals of equal length and left endpoints to compute a Riemann sum over the interval $[1, 2]$.

$$\text{Subinterval length:} \qquad \Delta x = \frac{2-1}{4} = \frac{1}{4} = 0.25$$

$$\begin{gathered}\text{Resulting partition:}\\ 1 < 1.25 < 1.5 < 1.75 < 2\end{gathered}$$

$$\begin{gathered}\text{Subinterval left endpoints:}\\ c_1 = 1, \qquad c_2 = 1.25, \qquad c_3 = 1.5, \qquad c_4 = 1.75\end{gathered}$$

$$\begin{gathered}\text{Riemann sum:}\\ \begin{aligned}\sum_{k=1}^{4} f(c_k)\Delta x \quad &= \quad 0.25 \cdot [f(1) + f(1.25) + f(1.5) + f(1.75)] \\ &= \quad 0.25 \cdot (0 + 0.1875 + 0.25 + 0.1875) \quad = \quad 0.15625\end{aligned}\end{gathered}$$