Math 131 - Test 1

September 9, 2026

Name Key

Score ________

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

1. (12 points) A shipping company uses the following formula to determine the cost (in dollars) of shipping a package that weighs x pounds:

$$C(x) = \begin{cases} 8.00, & x \leq 5 \\ 15.00 + 2.00(x-5), & x > 5 \end{cases}$$

(a) Use a table of numerical values to approximate $\lim_{x\to 5} C(x)$. Your table must show function values at six or more points.

x	4.9	4.99	4.999	5.1	5.01	5.001
$C(x)$	8	8	8	15.20	15.02	15.002

Looks like...

$\lim_{x\to 5^-} C(x) = 8$

$\lim_{x\to 5^+} C(x) = 15$

$\lim_{x\to 5} C(x)$ DNE

(b) Briefly explain why a customer who ships a package that weighs 5.01 lbs may feel ripped off by the company's pricing policy.

A 5-lb package ships for \$8, while a 5.01-lb package ships for \$15.02. That extra $\frac{1}{100}$ lb cost an extra \$7.02.

(c) Use any method to compute $\lim_{x\to 15} C(x)$. Show work or explain how you got your answer.

$$\lim_{x\to 15} C(x) = \lim_{x\to 15} \left[15 + 2(x-5)\right]$$

$$= 15 + 20 = 35$$

$\boxed{\$35}$

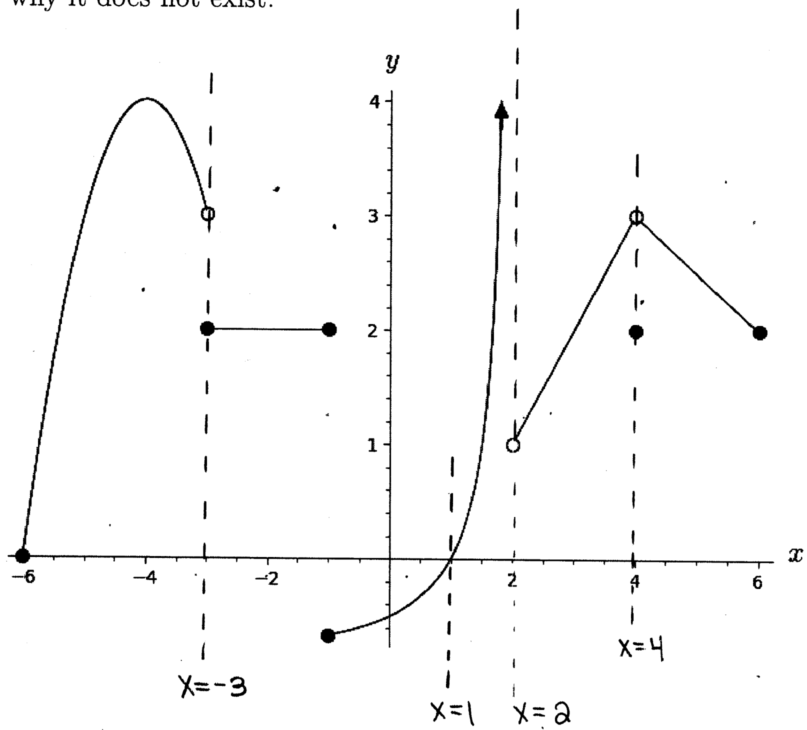
(d) Compute $C(8)$ and say what it represents.

$$C(8) = 15 + 2(8-5) = 15 + 6 = 21$$

It costs \$21 to ship an 8-lb package.

DIDN'T SHOW UP ON COPIES

2. (12 points) The graph of the function $y = f(x)$ is shown below. The dashed vertical line represents a vertical asymptote of the graph. Estimate each of the following or explain why it does not exist.



(a) $\lim_{x \to 1} f(x)$ = 0

(b) $\lim_{x \to -3} f(x)$ DNE. LIMIT FROM LEFT = 3 $\neq$ 2 = LIMIT FROM RIGHT

(c) $\lim_{x \to 4} f(x)$ = 3

(d) $f(4)$ = 2

(e) $\lim_{x \to 2^-} f(x)$ = $+\infty$ (BUT THE V.A. DIDN'T SHOW UP ON THE COPIES. :( )

(f) $\lim_{x \to 2^+} f(x)$ = 1

3. (9 points) These limits DO NOT EXIST. Carefully explain why each limit fails to exist. Show supporting work.

(a) $\lim_{x\to 1} \frac{x+1}{(x-1)^2}$

DIRECT SUBS GIVES A $\frac{2}{0}$ FORM. THIS FORM INDICATES THAT THE LIMIT DNE BECAUSE OF UNBOUNDED GROWTH AS $X \to 1$. IN FACT, $\lim_{X\to 1} \frac{X+1}{(X-1)^2} = +\infty$.

FAILURE #2

(b) $\lim_{x\to 0} x \ln x$

$\ln X$ IS NOT DEFINED WHEN $X < 0$. A TWO-SIDED LIMIT CANNOT EXIST.

FAILURE #4

(c) $\lim_{x\to 0} \cos\left(\frac{1}{x^2}\right)$

FAILURE #3

AS $X \to 0$, $\frac{1}{X^2} \to +\infty$. $\cos\left(\frac{1}{X^2}\right)$ WILL CYCLE THROUGH MORE & MORE PERIODS WITHOUT EVER APPROACHING A FIXED VALUE.

4. (6 points) Consider the limit $\lim_{x\to 0} x \sin\left(\frac{1}{x}\right)$.

(a) Briefly explain why direct substitution cannot be used to evaluate the limit.

DIRECT SUBS OF $X = 0$ WOULD RESULT IN DIVISION BY ZERO. DIVISION BY ZERO IS NOT DEFINED (BUT THAT SAYS NOTHING ABOUT THE LIMIT!).

(b) It is easy to prove that $-|x| \le x \sin\left(\frac{1}{x}\right) \le |x|$ for all $x \ne 0$. Evaluate $\lim_{x\to 0} x \sin\left(\frac{1}{x}\right)$ and explain your reasoning.

$\lim_{X\to 0} -|X| = \lim_{X\to 0} |X| = 0$. THEREFORE, $\lim_{X\to 0} X \sin\left(\frac{1}{X}\right) = 0$

BY SQUEEZE THEOREM.

5. (6 points) Determine a function $f(x)$ satisfying $\lim_{x\to 1}\left(\frac{f(x)}{x-1}\right) = 2$. After finding such a function, show that the limit is indeed 2.

LET $\boxed{f(x) = 2(X-1)}$. $\lim_{X\to 1} \frac{2\cancel{(X-1)}}{\cancel{X-1}} = \lim_{X\to 1} 2 = 2$

6. (24 points) **Determine each limit analytically** or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE. You will not be given credit if you get your answer from a table of values or a graph.

(a) $\lim_{x\to 2} \frac{3x^2 - 7x + 2}{x - 2}$ 0/0 MORE WORK

$$= \lim_{x\to 2} \frac{(3x-1)\cancel{(x-2)}}{\cancel{x-2}} = \lim_{x\to 2} (3x-1) = 3(2)-1 = \boxed{5}$$

(b) $\lim_{h\to 0} \frac{\sqrt{16+h} - 4}{h}$ 0/0 MORE WORK

$$\lim_{h\to 0} \frac{\sqrt{16+h} - 4}{h} \cdot \frac{\sqrt{16+h} + 4}{\sqrt{16+h} + 4} = \lim_{h\to 0} \frac{16+h-16}{h\,(\sqrt{16+h}+4)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{16+h}+4} = \boxed{\frac{1}{8}}$$

(c) $\lim_{r\to -2^-} \frac{|r+2|}{r^2 - 4}$ 0/0 MORE WORK

$|r+2|$
$= -(r+2)$
WHEN $r < -2$

$$= \lim_{r\to -2^-} \frac{-\cancel{(r+2)}}{\cancel{(r+2)}(r-2)} = \lim_{r\to -2^-} \frac{-1}{r-2} = \boxed{\frac{1}{4}}$$

(d) $\lim_{x\to 0+} f(x)$ where $f(x) = \begin{cases} \cos\left(\frac{1}{x}\right), & x \le 0 \\ \frac{\sin(5x)}{10x}, & x > 0 \end{cases}$

$$\lim_{x\to 0^+} \frac{\sin 5x}{10x} = \frac{1}{2} \lim_{x\to 0^+} \frac{\sin 5x}{5x}$$

$$= \frac{1}{2} \cdot 1 = \boxed{\frac{1}{2}}$$

7. (4 points) The *momentum* of a moving object of mass m and velocity x is given by

$$p(x) = \frac{mx}{\sqrt{1 - \frac{x^2}{c^2}}},$$

where c represents the speed of light. Find all vertical asymptotes of the graph of p.

NONZERO OVER ZERO FORM WHEN $1 - \frac{x^2}{c^2} = 0 \Rightarrow \boxed{x = \pm c}$

8. (9 points) For each part of this problem, **determine analytically** whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

(a) $\lim_{x \to 2} \frac{|x-1|}{|x-2|}$ $\quad 1/0$ SOME KIND OF UNBOUNDED GROWTH.

$\frac{|x-1|}{|x-2|}$ IS POSITIVE ON BOTH SIDES OF $x=2$ $\Rightarrow \lim_{x \to 2} \frac{|x-1|}{|x-2|} = \boxed{+\infty}$

(b) $\lim_{x \to 6^-} \frac{x}{x-6}$ $\quad 6/0$ SOME KIND OF UNBOUNDED GROWTH.

TO LEFT OF $x=6$...

$\frac{x}{x-6} = \frac{+}{-} = - \Rightarrow \boxed{\lim_{x \to 6^-} \frac{x}{x-6} = -\infty}$

(c) $\lim_{x \to 1} \frac{2x}{x-1}$ $\quad 2/0$ UNBOUNDED GROWTH.

LEFT OF $x=1$: $\frac{2x}{x-1} = \frac{+}{-} = - \Rightarrow \lim_{x \to 1^-} \frac{2x}{x-1} = -\infty$

RIGHT OF $x=1$: $\frac{2x}{x-1} = \frac{+}{+} = + \Rightarrow \lim_{x \to 1^+} \frac{2x}{x-1} = +\infty$

$\boxed{\text{LIMIT DNE.}}$

9. (8 points) Find the number β so that f is continuous at $x = 4$. For full credit, your work must show how you are using limits and the definition of continuity.

$$f(x) = \begin{cases} 7x + \cos(\pi x), & x < 4 \\ \beta x^2 - 5x + 9, & x \geq 4 \end{cases}$$

MUST HAVE $\lim_{x \to 4} f(x) = f(4)$.

$\lim_{x \to 4^-} f(x) = \lim_{x \to 4} (7x + \cos \pi x) = 28 + \cos(4\pi) = 29$,

$\lim_{x \to 4^+} f(x) = \lim_{x \to 4} (\beta x^2 - 5x + 9) = 16\beta - 11$

$f(4) = 16\beta - 11$

$\Longrightarrow$

$16\beta - 11 = 29$

$16\beta = 40$

$\beta = \frac{40}{16} = \frac{5}{2}$

$\boxed{\beta = \frac{5}{2}}$

10. (2 points) Which of these IS a reason that a limit may not exist?

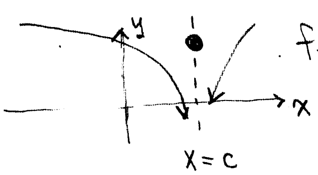
   (a) Direct substitution cannot be applied.
   (b) The function is not defined to the left of the limit point. ← FOR A TWO-SIDED LIMIT, FUNCTION MUST BE DEFINED ON BOTH SIDES OF LIMIT POINT.
   (c) The function is not defined at the limit point.
   (d) The function is not continuous at the limit point.

11. (2 points) Suppose you were asked to use a table of values to estimate $\lim_{x\to 2} f(x)$. Which list of x-values shown below would be best for your table?

   (a) $x = 1.9, 1.99, 1.999, 2.0, 2.1, 2.01, 2.001$ ← DON'T WANT X = 2 IN OUR TABLE
   (b) $x = 1.9, 1.99, 1.999, 2.1, 2.11, 2.111$ ← NOT GETTING CLOSER TO 2
   (c) $x = 2.00001, 2.000001, 2.0000001, 2.00000001, 2.000000001$ ← ONLY RIGHT SIDE OF 2
   (d) $x = 1.9, 1.99, 1.999, 2.1, 2.01, 2.001$

12. (2 points) Suppose $\lim_{x\to c} f(x) = -\infty$. Which one of the following is NOT necessarily true?

   (a) The graph of f has a vertical asymptote at $x = c$.
   (b) $\lim_{x\to c^+} f(x) = -\infty$
   (c) $\lim_{x\to c^-} f(x) = -\infty$
   (d) f is not defined at $x = c$.



f(c) IS DEFINED

13. (2 points) Suppose $\lim_{x\to 5} g(x) = 7$. Which one of these statements must be true?

   (a) g is continuous at $x = 5$.
   (b) g is defined at $x = 5$.
   (c) $g(5) = 7$
   (d) $\lim_{x\to 5^+} g(x) = 7$

BOTH ONE-SIDED LIMITS MUST EXIST AND BE EQUAL.

14. (2 points) Which one of the following best describes the meaning of the statement $\lim_{x\to 0} g(x) = \infty$?

   (a) Direct substitution results in division by zero.
   (b) The limit at $x = 0$ exists, and it is a very large positive number.
   (c) The limit at $x = 0$ does not exist because $g(0)$ is not defined.
   (d) The limit at $x = 0$ does not exist because the values of g grow positively without bound as $x \to 0$.