

Math 131 - Quiz 4

September 16, 2026

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (3 points) Determine if f is continuous at $x = 0$. If not, classify the discontinuity. For full credit, your work must show how you are using limits and the definition of continuity.

$$f(x) = \begin{cases} x^2 - e^x, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$$

$$\left. \begin{array}{l} f(0) = 0 - 1 = -1 \\ \lim_{x \rightarrow 0^-} f(x) = 0^2 - e^0 = -1 \\ \lim_{x \rightarrow 0^+} f(x) = 0 - 1 = -1 \end{array} \right\} \quad \lim_{x \rightarrow 0} f(x) = f(0) \Rightarrow f \text{ is continuous at } x = 0.$$

2. (3 points) Find the value of k that makes f continuous on its domain. For full credit, your work must show how you are using limits and the definition of continuity.

$$f(x) = \begin{cases} 3x + 2, & x < k \\ 2x - 3, & k \leq x \leq 8 \end{cases} \quad \left. \begin{array}{l} f(k) = 2k - 3 \\ \lim_{x \rightarrow k^-} f(x) = 3k + 2 \\ \lim_{x \rightarrow k^+} f(x) = 2k - 3 \end{array} \right\} \begin{array}{l} \text{Must} \\ \text{be} \\ \text{equal} \end{array}$$

$$3k + 2 = 2k - 3$$

$$\boxed{k = -5}$$

Only possible discontinuity is at $x = k$.

3. (2 points) Find and classify all discontinuities of $f(x) = \frac{\sin x}{x}$.

The only discont is $x = 0$

And it is removable because $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.

Limit exists!

4. (2 points) Use the Intermediate Value Theorem (IVT) to find an interval of length one (or less) on which $x^5 + 4x + 1 = 0$ has a solution. Your work must show how you used the IVT.

 $f(x)$ is continuous.

$$\begin{array}{l} f(-1) = -4 < 0 \\ f(0) = 1 > 0 \end{array} \Rightarrow f(x) = 0 \text{ between } x = -1 \text{ and } x = 0.$$