

Math 131 - Test 1
September 10, 2026

Name _____

Score _____

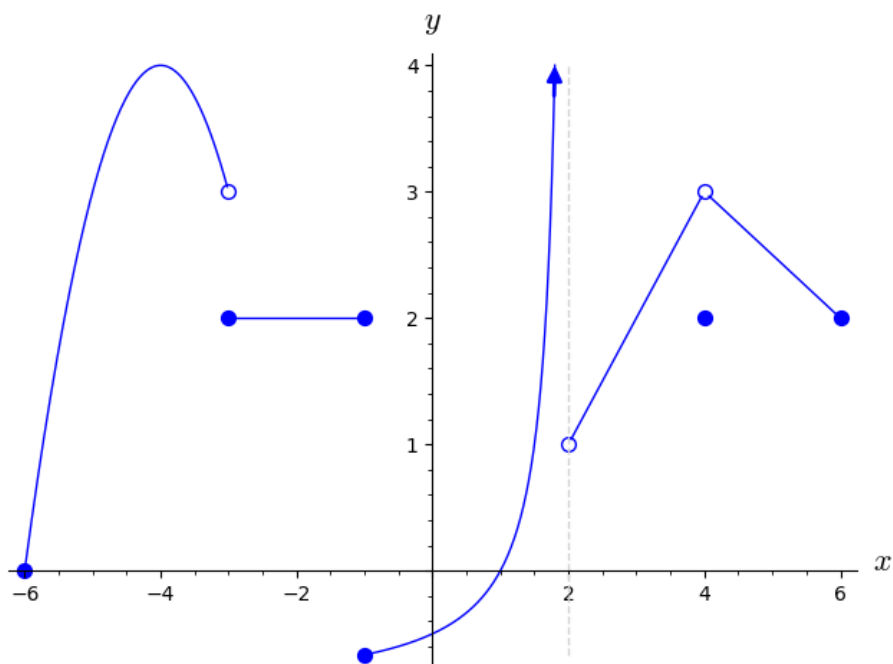
Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

1. (12 points) A shipping company uses the following formula to determine the cost (in dollars) of shipping a package that weighs x pounds:

$$C(x) = \begin{cases} 8.00, & x \leq 5 \\ 15.00 + 2.00(x - 5), & x > 5 \end{cases}$$

- (a) Use a table of numerical values to approximate $\lim_{x \rightarrow 5} C(x)$. Your table must show function values at six or more points.
- (b) Briefly explain why a customer who ships a package that weighs 5.01 lbs may feel ripped off by the company's pricing policy.
- (c) Use any method to compute $\lim_{x \rightarrow 15} C(x)$. Show work or explain how you got your answer.
- (d) Compute $C(8)$ and say what it represents.

2. (12 points) The graph of the function $y = f(x)$ is shown below. The dashed vertical line represents a vertical asymptote of the graph. Estimate each of the following or explain why it does not exist.



(a) $\lim_{x \rightarrow 1} f(x)$

(b) $\lim_{x \rightarrow -3} f(x)$

(c) $\lim_{x \rightarrow 4} f(x)$

(d) $f(4)$

(e) $\lim_{x \rightarrow 2^-} f(x)$

(f) $\lim_{x \rightarrow 2^+} f(x)$

3. (9 points) These limits DO NOT EXIST. Carefully explain why each limit fails to exist. Show supporting work.

(a) $\lim_{x \rightarrow 1} \frac{x+1}{(x-1)^2}$

(b) $\lim_{x \rightarrow 0} x \ln x$

(c) $\lim_{x \rightarrow 0} \cos\left(\frac{1}{x^2}\right)$

4. (6 points) Consider the limit $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$.

(a) Briefly explain why direct substitution cannot be used to evaluate the limit.

(b) It is easy to prove that $-|x| \leq x \sin\left(\frac{1}{x}\right) \leq |x|$ for all $x \neq 0$. Evaluate $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$ and explain your reasoning.

5. (6 points) Determine a function $f(x)$ satisfying $\lim_{x \rightarrow 1} \left(\frac{f(x)}{x-1}\right) = 2$. After finding such a function, show that the limit is indeed 2.

6. (24 points) **Determine each limit analytically** or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE. You will not be given credit if you get your answer from a table of values or a graph.

(a) $\lim_{x \rightarrow 2} \frac{3x^2 - 7x + 2}{x - 2}$

(b) $\lim_{h \rightarrow 0} \frac{\sqrt{16 + h} - 4}{h}$

(c) $\lim_{r \rightarrow -2^-} \frac{|r + 2|}{r^2 - 4}$

(d) $\lim_{x \rightarrow 0^+} f(x)$ where $f(x) = \begin{cases} \cos\left(\frac{1}{x}\right), & x \leq 0 \\ \frac{\sin(5x)}{10x}, & x > 0 \end{cases}$

7. (4 points) The *momentum* of a moving object of mass m and velocity x is given by

$$p(x) = \frac{mx}{\sqrt{1 - \frac{x^2}{c^2}}},$$

where c represents the speed of light. Find all vertical asymptotes of the graph of p .

8. (9 points) For each part of this problem, **determine analytically** whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

(a) $\lim_{x \rightarrow 2} \frac{|x - 1|}{|x - 2|}$

(b) $\lim_{x \rightarrow 6^-} \frac{x}{x - 6}$

(c) $\lim_{x \rightarrow 1} \frac{2x}{x - 1}$

9. (8 points) Find the number β so that f is continuous at $x = 4$. For full credit, your work must show how you are using limits and the definition of continuity.

$$f(x) = \begin{cases} 7x + \cos(\pi x), & x < 4 \\ \beta x^2 - 5x + 9, & x \geq 4 \end{cases}$$

10. (2 points) Which of these IS a reason that a limit may not exist?
- (a) Direct substitution cannot be applied.
 - (b) The function is not defined to the left of the limit point.
 - (c) The function is not defined at the limit point.
 - (d) The function is not continuous at the limit point.
11. (2 points) Suppose you were asked to use a table of values to estimate $\lim_{x \rightarrow 2} f(x)$. Which list of x -values shown below would be best for your table?
- (a) $x = 1.9, 1.99, 1.999, 2.0, 2.1, 2.01, 2.001$
 - (b) $x = 1.9, 1.99, 1.999, 2.1, 2.11, 2.111$
 - (c) $x = 2.00001, 2.000001, 2.0000001, 2.00000001, 2.000000001$
 - (d) $x = 1.9, 1.99, 1.999, 2.1, 2.01, 2.001$
12. (2 points) Suppose $\lim_{x \rightarrow c} f(x) = -\infty$. Which one of the following is NOT necessarily true?
- (a) The graph of f has a vertical asymptote at $x = c$.
 - (b) $\lim_{x \rightarrow c^+} f(x) = -\infty$
 - (c) $\lim_{x \rightarrow c^-} f(x) = -\infty$
 - (d) f is not defined at $x = c$.
13. (2 points) Suppose $\lim_{x \rightarrow 5} g(x) = 7$. Which one of these statements must be true?
- (a) g is continuous at $x = 5$.
 - (b) g is defined at $x = 5$.
 - (c) $g(5) = 7$
 - (d) $\lim_{x \rightarrow 5^+} g(x) = 7$
14. (2 points) Which one of the following best describes the meaning of the statement $\lim_{x \rightarrow 0} g(x) = \infty$?
- (a) Direct substitution results in division by zero.
 - (b) The limit at $x = 0$ exists, and it is a very large positive number.
 - (c) The limit at $x = 0$ does not exist because $g(0)$ is not defined.
 - (d) The limit at $x = 0$ does not exist because the values of g grow positively without bound as $x \rightarrow 0$.