Math 131 - Test 1

September 9, 2026

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

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1. (12 points) A shipping company uses the following formula to determine the cost (in dollars) of shipping a package that weighs x pounds:

$$C(x) = \begin{cases} 8.00, & x \leq 5 \\ 15.00 + 2.00(x-5), & x > 5 \end{cases}$$

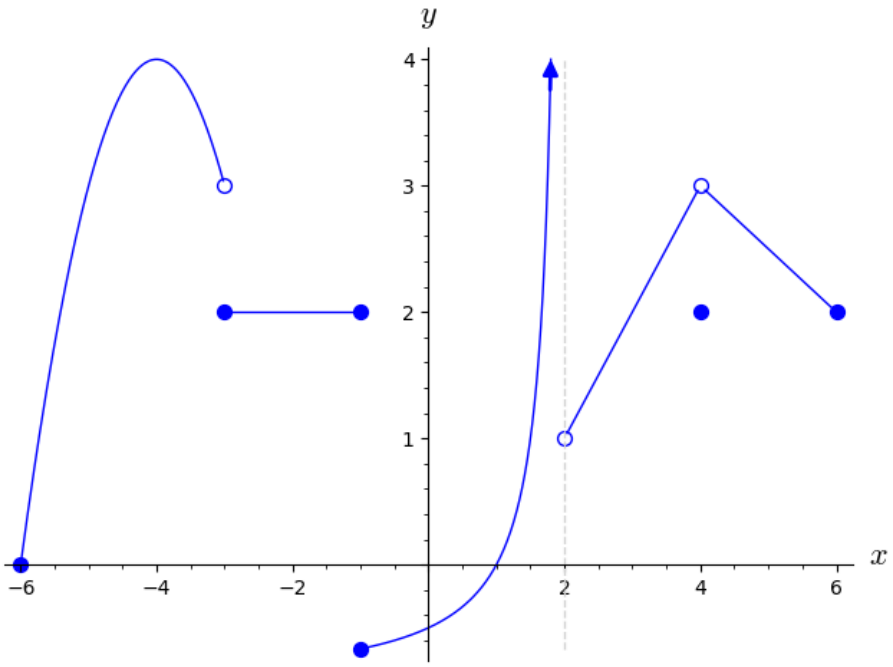
   (a) Use a table of numerical values to approximate $\lim_{x \to 5} C(x)$. Your table must show function values at six or more points.

   (b) Briefly explain why a customer who ships a package that weighs 5.01 lbs may feel ripped off by the company's pricing policy.

   (c) Use any method to compute $\lim_{x \to 15} C(x)$. Show work or explain how you got your answer.

   (d) Compute $C(8)$ and say what it represents.

2. (12 points) The graph of the function $y = f(x)$ is shown below. The dashed vertical line represents a vertical asymptote of the graph. Estimate each of the following or explain why it does not exist.



(a) $\lim_{x \to 1} f(x)$

(b) $\lim_{x \to -3} f(x)$

(c) $\lim_{x \to 4} f(x)$

(d) $f(4)$

(e) $\lim_{x \to 2^-} f(x)$

(f) $\lim_{x \to 2^+} f(x)$

3. (9 points) These limits DO NOT EXIST. Carefully explain why each limit fails to exist. Show supporting work.

   (a) $\lim_{x\to 1} \dfrac{x+1}{(x-1)^2}$

   (b) $\lim_{x\to 0} x \ln x$

   (c) $\lim_{x\to 0} \cos\left(\dfrac{1}{x^2}\right)$

4. (6 points) Consider the limit $\lim_{x\to 0} x \sin\left(\dfrac{1}{x}\right)$.

   (a) Briefly explain why direct substitution cannot be used to evaluate the limit.

   (b) It is easy to prove that $-|x| \leq x \sin\left(\dfrac{1}{x}\right) \leq |x|$ for all $x \neq 0$. Evaluate $\lim_{x\to 0} x \sin\left(\dfrac{1}{x}\right)$ and explain your reasoning.

5. (6 points) Determine a function $f(x)$ satisfying $\lim_{x\to 1} \left(\dfrac{f(x)}{x-1}\right) = 2$. After finding such a function, show that the limit is indeed 2.

6. (24 points) **Determine each limit analytically** or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE. You will not be given credit if you get your answer from a table of values or a graph.

(a) $\lim_{x\to 2} \dfrac{3x^2 - 7x + 2}{x - 2}$

(b) $\lim_{h\to 0} \dfrac{\sqrt{16+h} - 4}{h}$

(c) $\lim_{r\to -2^-} \dfrac{|r+2|}{r^2 - 4}$

(d) $\lim_{x\to 0+} f(x)$ where $f(x) = \begin{cases} \cos\left(\dfrac{1}{x}\right), & x \le 0 \\ \dfrac{\sin(5x)}{10x}, & x > 0 \end{cases}$

7. (4 points) The *momentum* of a moving object of mass m and velocity x is given by

$$p(x) = \frac{mx}{\sqrt{1 - \dfrac{x^2}{c^2}}},$$

where c represents the speed of light. Find all vertical asymptotes of the graph of p.

8. (9 points) For each part of this problem, **determine analytically** whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

(a) $\lim\limits_{x \to 2} \dfrac{|x-1|}{|x-2|}$

(b) $\lim\limits_{x \to 6^-} \dfrac{x}{x-6}$

(c) $\lim\limits_{x \to 1} \dfrac{2x}{x-1}$

9. (8 points) Find the number β so that f is continuous at $x = 4$. For full credit, your work must show how you are using limits and the definition of continuity.

$$f(x) = \begin{cases} 7x + \cos(\pi x), & x < 4 \\ \beta x^2 - 5x + 9, & x \geq 4 \end{cases}$$

10. (2 points) Which of these IS a reason that a limit may not exist?

    (a) Direct substitution cannot be applied.

    (b) The function is not defined to the left of the limit point.

    (c) The function is not defined at the limit point.

    (d) The function is not continuous at the limit point.

11. (2 points) Suppose you were asked to use a table of values to estimate $\lim_{x\to 2} f(x)$. Which list of x-values shown below would be best for your table?

    (a) $x = 1.9,\ 1.99,\ 1.999,\ 2.0,\ 2.1,\ 2.01,\ 2.001$

    (b) $x = 1.9,\ 1.99,\ 1.999,\ 2.1,\ 2.11,\ 2.111$

    (c) $x = 2.00001,\ 2.000001,\ 2.0000001,\ 2.00000001,\ 2.000000001$

    (d) $x = 1.9,\ 1.99,\ 1.999,\ 2.1,\ 2.01,\ 2.001$

12. (2 points) Suppose $\lim_{x\to c} f(x) = -\infty$. Which one of the following is NOT necessarily true?

    (a) The graph of f has a vertical asymptote at $x = c$.

    (b) $\lim_{x\to c^+} f(x) = -\infty$

    (c) $\lim_{x\to c^-} f(x) = -\infty$

    (d) f is not defined at $x = c$.

13. (2 points) Suppose $\lim_{x\to 5} g(x) = 7$. Which one of these statements must be true?

    (a) g is continuous at $x = 5$.

    (b) g is defined at $x = 5$.

    (c) $g(5) = 7$

    (d) $\lim_{x\to 5^+} g(x) = 7$

14. (2 points) Which one of the following best describes the meaning of the statement $\lim_{x\to 0} g(x) = \infty$?

    (a) Direct substitution results in division by zero.

    (b) The limit at $x = 0$ exists, and it is a very large positive number.

    (c) The limit at $x = 0$ does not exist because $g(0)$ is not defined.

    (d) The limit at $x = 0$ does not exist because the values of g grow positively without bound as $x \to 0$.