Math 131 - Quiz 12

December 4, 2025

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due December 9.

1. (3 points) Evaluate each limit.

(a) $\lim_{t\to 0}\left(\frac{t\sin t}{1-\cos t}\right)$ $0/0$

$$= \lim_{t\to 0} \frac{\sin t + t\cos t}{\sin t} \overset{0/0}{=} \lim_{t\to 0} \frac{\cos t + \cos t - t\sin t}{\cos t} = \frac{1+1-0}{1} = \boxed{2}$$

(b) $\lim_{x\to\infty} x^2 e^{-x}$ $\infty\cdot 0$

$$= \lim_{x\to\infty} \frac{x^2}{e^x} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{2x}{e^x} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{2}{e^x} = \boxed{0}$$

(c) $\lim_{x\to 0^+}\left(\frac{3x+1}{x} - \frac{1}{\sin x}\right)$ $\infty - \infty$ $= \lim_{x\to 0^+} \frac{(3x+1)\sin x - x}{x\sin x}$ $0/0$

$$= \lim_{x\to 0^+} \frac{3\sin x + (3x+1)\cos x - 1}{\sin x + x\cos x} \overset{0/0}{=} \lim_{x\to 0^+} \frac{3\cos x + 3\cos x - (3x+1)\sin x}{\cos x + \cos x - x\sin x}$$

$$= \frac{3+3-0}{1+1-0} = \frac{6}{2} = \boxed{3}$$

2. (1 point) Find the function f that satisfies $f'(x) = \frac{8}{1+x^2} + \sec^2 x$, $f(0) = 1$.

$$f(x) = 8\tan^{-1}x + \tan x + C$$

$$f(0) = 1 \Rightarrow 8\tan^{-1}(0) + \tan(0) + C$$
$$= 0 + 0 + C = 1 \Rightarrow C = 1$$

$$\boxed{f(x) = 8\tan^{-1}x + \tan x + 1}$$

Turn over.

3. (3 points) Evaluate each indefinite integral.

(a) $\int (x^2 + x^3 + x^{\sqrt{3}})\, dx$

$$= \boxed{\frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{1+\sqrt{3}}\, x^{1+\sqrt{3}} + C}$$

(b) $\int (\sqrt[5]{x} + \sqrt{x} - 5\sin x)\, dx$

$$= \int \left(x^{1/5} + x^{1/2} - 5\sin x\right) dx = \boxed{\frac{5}{6}x^{6/5} + \frac{2}{3}x^{3/2} + 5\cos x + C}$$

(c) $\int \left(\frac{1}{x} + \frac{1}{x^2}\right) dx \;=\; \int \left(\frac{1}{x} + x^{-2}\right) dx$

$$= \ln|x| + \frac{x^{-1}}{-1} + C = \boxed{\ln|x| - \frac{1}{x} + C}$$

4. (1.5 points) Let $f(x) = \sin(\pi x)$. Use 4 subintervals of equal length and subinterval midpoints to compute a Riemann sum for f on $[1, 3]$.

$\Delta x = 0.5$

PARTITION: $1 < 1.5 < 2 < 2.5 < 3$

$c_1 = 1.25,\ c_2 = 1.75,\ c_3 = 2.25$

$c_4 = 2.75$

RIEMANN SUM $= 0.5\left[\sin(1.25\pi) + \sin(1.75\pi) + \sin(2.25\pi) + \sin(2.75\pi)\right]$

$$= 0.5\left[-\frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right]$$

$$= \boxed{0}$$

5. (1.5 points) Let $f(x) = e^x$. Use 5 subintervals of equal length and subinterval left endpoints to compute a Riemann sum for f on $[0, 1]$.

$\Delta x = 0.2$

PARTITION: $0 < 0.2 < 0.4 < 0.6 < 0.8 < 1$

$$\underset{c_1}{0} \quad \underset{c_2}{0.2} \quad \underset{c_3}{0.4} \quad \underset{c_4}{0.6} \quad \underset{c_5}{0.8}$$

RIEMANN SUM $=$

$$0.2\left[e^0 + e^{0.2} + e^{0.4} + e^{0.6} + e^{0.8}\right]$$

$$\approx 0.2\left[7.760887\right]$$

$$\approx \boxed{1.552}$$