

Math 131 - Test 2

October 9, 2024

Name _____

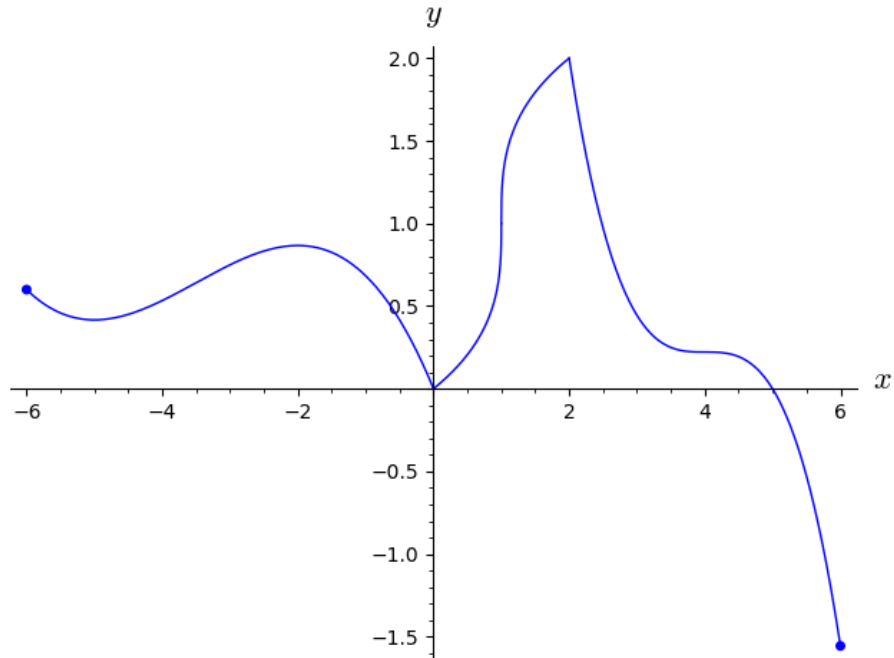
Score _____

Show all work to receive full credit. Supply explanations where necessary. Unless otherwise indicated, use differentiation rules for all derivatives and do not simplify.

1. (9 points) Let $f(x) = 3x^2 - x$. Use the limit definition of the derivative to compute $f'(x)$. Show all work.

2. (3 points) Using your result from above, find the instantaneous rate of change of f at the point where $x = 2$.

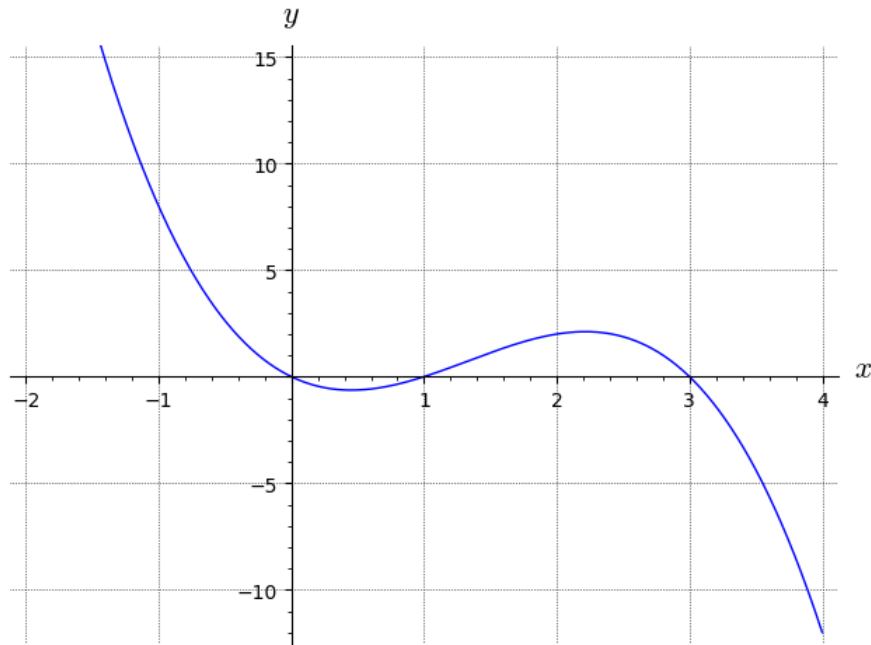
3. (12 points) Use the graph of $y = f(x)$ is shown below. Use the graph to solve each part of this problem.



- (a) Find three x -values in the interval $(-6, 6)$ at which $f'(x)$ does not exist. Explain why $f'(x)$ DNE at each of your points.
- (b) Determine an x -value at which $f'(x) = 0$. Explain how you know.
- (c) Sketch the tangent line at the point where $x = -4$. Then use your tangent line to estimate $f'(-4)$. Show work or explain your reasoning.
- (d) Determine an x -value at which $f'(x) < 0$. Explain how you know.

4. (6 points) The graph of the function $y = f(x)$ is shown below. By using the graph to estimate them, place the following values in order from least to greatest. Explain or show work.

$$f'(1), \quad f'(-1), \quad f(1), \quad f'(2)$$



5. (6 points) Find an equation of the tangent line to the graph of $y = x^3 + \sqrt[3]{x} + \frac{1}{x^3}$ at the point where $x = -1$. Write your final answer in slope-intercept form.

6. (12 points) The following table gives information about the functions f and g and their derivatives at selected points.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	6	0	-6	-3
1	7	2	-3	-1
2	-7	4	0	1
3	-4	8	1	5

(a) Find $h'(1)$ if $h(x) = 2x^2f(x)$.

(b) Find $h'(3)$ if $h(x) = f(x)/g(x)$.

(c) Find $h'(0)$ if $h(x) = [g(x)]^4$.

(d) Find $h'(2)$ if $h(x) = f(g(x))$.

7. (4 points) Use trig identities and the quotient rule to derive our formula for the derivative of $y = \tan x$ from the basic rules for the sine and cosine.

8. (12 points) A potato is launched vertically upward with an initial velocity of 48 ft/s from a potato gun at the top of an 160-foot-tall building. After t seconds, the potato's height (in feet) measured from the ground is given by $s(t) = -16t^2 + 48t + 160$.
- (a) Determine the maximum height of the potato.
- (b) Determine when the potato hits the ground.
- (c) Determine the speed of the potato when it hits the ground.
9. (5 points) Let $w(x) = x^5 + x^2 - 7 \sin x$. Find $w'''(x)$.
10. (7 points) Let $F(x) = \tan^3(x^2 + x + 1)$. Compute $F'(x)$. (You'll need to use the chain rule twice.)

11. (15 points) Differentiate. Do not simplify.

(a) $\frac{d}{dx}[x^4(x^2 + 2x + 4)]$

(b) $\frac{d}{dx} \left(\frac{x \cos x}{2x + 5} \right)$

(c) $\frac{d}{dt} \sqrt{t^3 + 2t + 1}$

12. (9 points) Assume that y is implicitly defined as a function of x by the equation $x^4y + y^3 = -2x$. Use implicit differentiation to find dy/dx at $(-1, 1)$. Then find an equation for the normal line at $(-1, 1)$.