Math 131 - Assignment 8

October 24, 2024

Name Key

Score

Show all work to receive full credit. Supply explanations when necessary. This assignment is due October 31.

1. Let $f(x) = \frac{1}{x} + \sqrt[3]{x}$.

   (a) Determine the linearization of f at $x = 8$. Write your answer in exact form (fractions, not decimals).

$f(8) = \frac{1}{8} + 2 = \frac{17}{8}$

$\boxed{L(x) = \frac{17}{8} + \frac{13}{192}(x-8)}$

$f'(x) = -\frac{1}{x^2} + \frac{1}{3}x^{-2/3}$ $\quad f'(8) = -\frac{1}{64} + \frac{1}{12} = \frac{13}{192}$

   (b) Use your linearization to approximate $f(8.1)$. Round to the 6th decimal place.

$f(8.1) \approx L(8.1) = \frac{17}{8} + \frac{13}{192}(0.1) = \boxed{\frac{4093}{1920} \approx 2.131771}$

2. Some values of $f(x)$ and $f'(x)$ near $x = 1$ are given in the table below.

x	0.50	0.75	1.00	1.25	1.50
$f(x)$	6.08	6.90	8.00	9.41	11.14
$f'(x)$	2.74	3.82	5.00	6.26	7.60

   (a) Determine the linearization of f at $x = 1$.

$L(x) = f(1) + f'(1)(x-1) \Rightarrow \boxed{L(x) = 8.00 + 5.00(x-1)}$

$= 5x + 3$

   (b) Use the linearization you found above to approximate $f(0.75)$.

$f(0.75) \approx L(0.75) = 5(0.75) + 3$

$= \boxed{6.75}$

Turn over.

$f'(x) = 2x + \frac{1}{2}x^{-1/2} - \frac{1}{x^2}$

3. Find the linearization of $f(x) = x^2 + x^{1/2} + \dfrac{1}{x}$ at $x = 1$, then use it to approximate $f(0.98)$.

$f(1) = 3$

$\boxed{L(x) = 3 + \frac{3}{2}(x-1)}$

$f'(1) = 2 + \frac{1}{2} - 1$
$= \frac{3}{2}$

$f(0.98) \approx L(0.98) = 3 + \frac{3}{2}(-0.02)$
$= \boxed{2.97}$

4. Use differentials to approximate the change in $y = \sqrt{x^3+1}$ as x changes from 2 to 2.07.

$\Delta y \approx f'(x)\,\Delta x$

$x = 2$
$\Delta x = 0.07$

$\Delta y \approx 2(0.07)$
$= \boxed{0.14}$

$f'(x) = \frac{1}{2}(x^3+1)^{-1/2}(3x^2)$

$f'(2) = \frac{1}{2}\left(\frac{1}{3}\right)(12) = 2$

5. Determine the differential dy.

(a) $y = 5^{x^2+1}$

$\frac{dy}{dx} = 5^{x^2+1}(\ln 5)(2x) \Rightarrow \boxed{dy = (5^{x^2+1})(\ln 5)(2x)\,dx}$

(b) $y = \cot^{-1}(\sqrt{x})$

$\frac{dy}{dx} = \frac{-1}{1+x} \cdot \frac{1}{2}x^{-1/2} \Longrightarrow \boxed{dy = \frac{-1}{2\sqrt{x}\,(1+x)}\,dx}$

6. Use differentials to approximate the change in $y = \dfrac{1}{1-x}$ as x changes from 2 to 1.98.

$\Delta y \approx \frac{dy}{dx}\Delta x$

$x = 2$
$\Delta x = -0.02$

$\Delta y \approx (1)(-0.02)$
$= \boxed{-0.02}$

$\frac{dy}{dx} = \frac{1}{(1-x)^2}$

$\left.\frac{dy}{dx}\right|_{x=2} = 1$

7. Suppose that the percent error in measuring the side length of a cube is 2%. Use differentials to estimate the percent error in computing the cube's volume.

$V = x^3$ WHERE x = SIDE LENGTH

$\Delta V \approx 3x^2\Delta x \Rightarrow \frac{\Delta V}{V} = \frac{3x^2\Delta x}{x^3} = \frac{3}{x}\Delta x = \frac{3}{x}(0.02x) = 0.06 = \boxed{6\%}$