Math 131 - Test 2

October 12, 2022

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. Unless otherwise indicated, use differentiation rules for all derivatives and do not simplify.

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1. (10 points) Let $f(x) = \sqrt{x}$. Use a **limit definition of the derivative** to determine $f'(x)$. Show all work.

2. (4 points) Which one of the following best describes the line tangent to the graph of $g(x) = |2x - 6|$ at the point $(3, 0)$? (Briefly explain or show work to receive full credit.)

   (a) The tangent line is horizontal.

   (b) The tangent line is vertical.

   (c) A unique tangent line does not exist.

   (d) The tangent line has slope 1.

3. (4 points) Which one of the following best describes the line tangent to the graph of $f(x) = 5x^{1/3} - 2$ at the point $(0, -2)$? (Briefly explain or show work to receive full credit.)

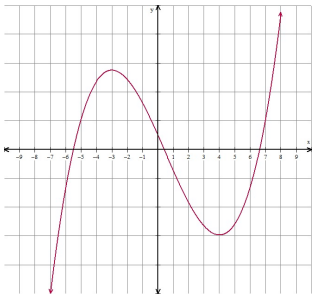
   (a) The tangent line is horizontal.

   (b) The tangent line is vertical.

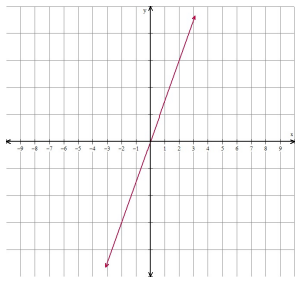
   (c) A unique tangent line does not exist.

   (d) The tangent line has slope -2.

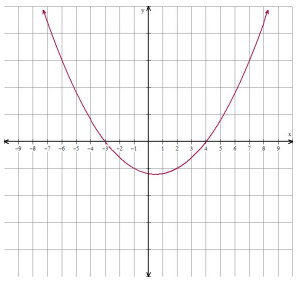
4. (6 points) The graph of $g(x)$ is shown below. Choose the lettered graph that best represents the graph of $g'(x)$. Explain your reasoning. Give at least two reasons to support your answer.



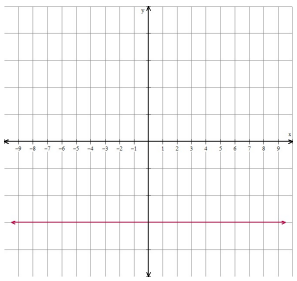
A



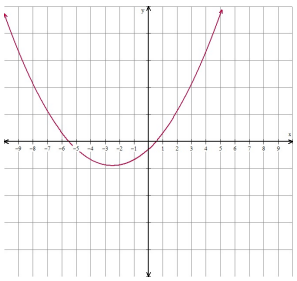
B



C



D



5. An object is launched vertically so that its height (in feet) after t seconds is given by

$$s(t) = -16t^2 + 72t + 40.$$

Include units with your answer for each part of this problem.

(a) (3 points) Determine the average rate of change the object's height over the interval from $t = 0$ to $t = 2$.

(b) (3 points) Determine the object's velocity at time $t = 3$.

(c) (2 points) What is the acceleration of the object?

(d) (4 points) Determine the object's maximum height.

(e) (3 points) When does the object hit the ground?

(f) (1 point) What is the object's initial speed?

(g) (1 point) What is the object's speed when it hits the ground?

6. (20 points) Differentiate. Do not simplify.

(a) $\dfrac{d}{dt}\left(10 + \dfrac{2}{t} - \dfrac{1}{t^2} + \sqrt[5]{t^3}\right)$

(b) $\dfrac{d}{dx}\left(\dfrac{4x^5 - 7x^2}{\csc x}\right)$

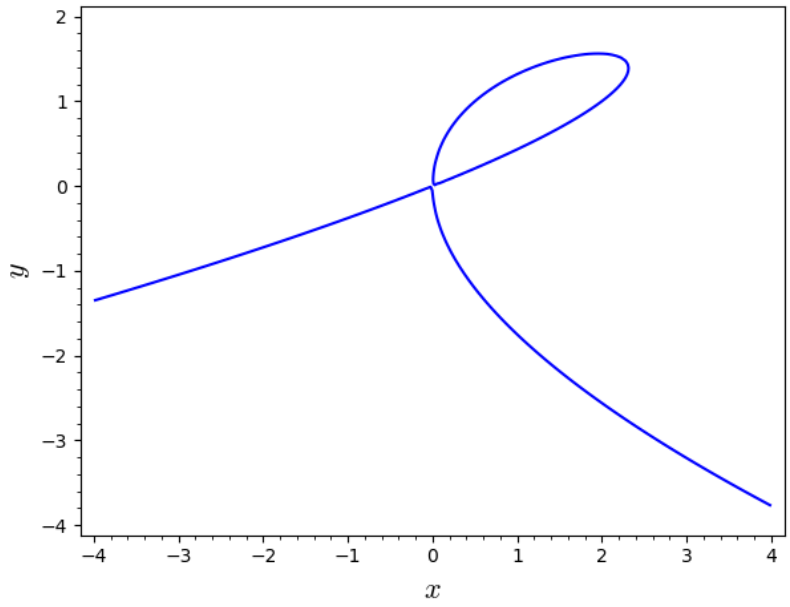
(c) $\dfrac{d}{dx}\sec(\sqrt{x})$

(d) $\dfrac{d}{dt}\left[(8t-3)^5 \sin(t^2)\right]$

7. (6 points) Let $f(x) = 8x\cos x$. Find $f''(x)$.

8. (4 points) The line $y = 2x + 3$ is tangent to a graph at the point $(3, 9)$. Find an equation for the line normal to the graph at $(3, 9)$.

9. (12 points) The graph of the equation $x^2 + y^3 = \frac{5}{2}xy$ is shown below.

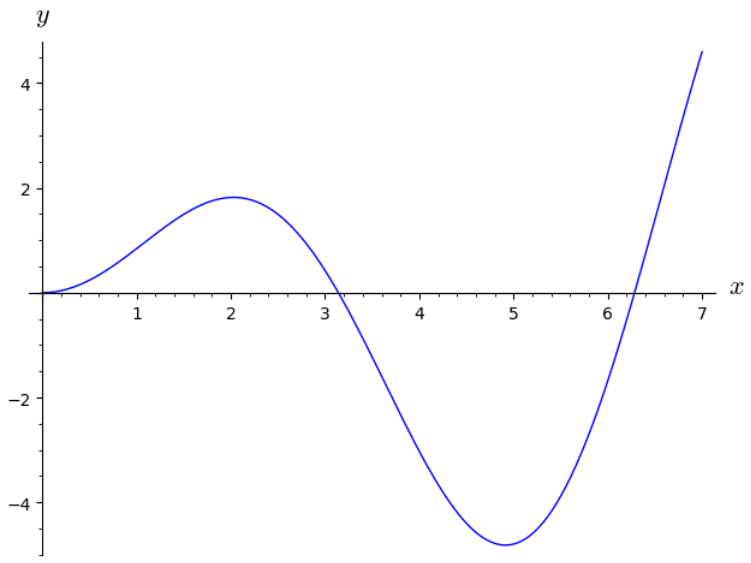


(a) Use implicit differentiation to find a formula for dy/dx.

(b) Use dy/dx to compute the slope of the graph at the point $(2, 1)$. Then determine an equation for the tangent line at $(2, 1)$.

10. (8 points) The graph of the function f is shown below. Referring to the graph, place the following values in order from least to greatest. Explain or show work to receive full credit.

$$f'(1), \quad f'(5), \quad f(5), \quad f(7), \quad f'(3)$$



11. (6 points) Let $f(x) = x^5 + x^3 - 30$. Compute $(f^{-1})'(10)$.

12. (3 points) Let $g(x) = 7x + 5$. Determine $(g^{-1})'(x)$.