Math 131 - Test 1

Name _____

September 15, 2021

Score _____

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist). When classifying discontinuities, use the words *removable*, *nonremovable*, *infinite*, and/or *jump*.

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1. (10 points) Think about our definition of *limit*. For each part below, does the table of values justify the given limit (yes or no)? Explain your reasoning.

   (a) $\lim_{x \to 2} f(x) = 1$

x	2.1	2.01	2.001	2.0001	2.00001	2.000001
$f(x)$	1.098363	1.009998	1.001000	1.000100	1.000010	1.000001

   (b) $\lim_{x \to 0} g(x) = 7$

x	0.01	-0.01	0.001	-0.001	0.0001	-0.0001
$g(x)$	2.976341	3.001253	4.994369	5.000124	6.999998	7.000002

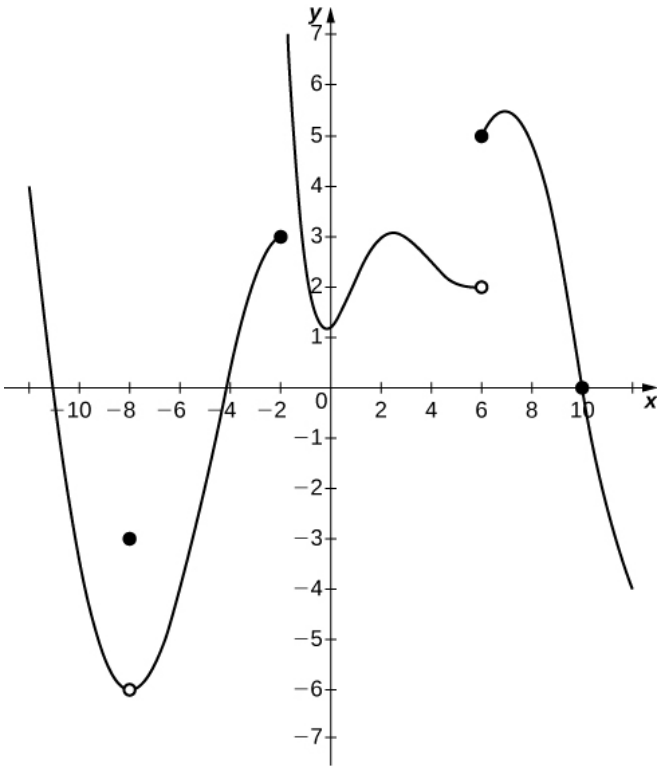
   (c) $\lim_{x \to 3.5} h(x) = 8.5$

x	1.00	2.00	3.00	4.00	5.00	6.00
$h(x)$	8.342301	8.497634	8.499436	8.500293	8.522341	8.299863

   (d) $\lim_{x \to 4^+} f(x) = \infty$

x	3.9	3.99	3.999	3.9999	3.99999	3.999999
$f(x)$	15.3	135.7	12,365.8	1,302,986.4	186,732,001.5	2,332,986,094.3

2. (14 points) The graph of $y = f(x)$ is shown below. Use the graph to solve each part of this problem.



(a) What type of discontinuity does f have at $x = 6$? Explain your reasoning.

(b) Estimate $\lim_{x \to 10^+} f(x)$.

(c) Estimate the value $f(-8)$.

(d) Based on the graph, Steve believed that $\lim_{x \to -2^-} f(x) = \infty$. Do you agree or disagree? Explain your reasoning.

(e) Estimate $\lim_{x \to -6} f(x)$.

(f) What type of discontinuity does f have at $x = -8$? Explain your reasoning.

(g) Estimate $\lim_{x \to 6^-} f(x)$.

3. (30 points) Determine each limit **analytically**, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE.

(a) $\lim_{w\to 6} \dfrac{\sqrt{w+3}-3}{2w-12}$

(b) $\lim_{x\to 2^+} \dfrac{x^2+7x+10}{x^2+3x+2}$

(c) $\lim_{h\to -5} \left(\dfrac{\frac{3}{h}+\frac{3}{5}}{h+5}\right)$

(d) $\lim_{y\to 0} \dfrac{y}{(y+6)^2-36}$

(e) $\lim_{x\to 0} \dfrac{\tan 3x}{6x}$

4. (12 points) These limits DO NOT EXIST. **Choose any three (3) of them**, and clearly tell why the limit fails to exist. If necessary, provide evidence.

   (a) $\lim_{x\to 7} \dfrac{9x}{(x-7)^4}$

   (b) $\lim_{x\to 0} x^2 \ln x$

   (c) $\lim_{x\to 0} \dfrac{x^2+3x}{|x|}$

   (d) $\lim_{x\to \pi^+} \left(\dfrac{x-3}{\tan x}\right)$

5. (9 points) Suppose that $\lim_{x\to 3} f(x) = 4$ and $\lim_{x\to 3} h(x)$ exists. Determine each limit.

   (a) $\lim_{x\to 3} [x^2 f(x) + h(x)\sin \pi x]$

   (b) $\lim_{x\to 3} [(x-3)f(x)h(x)]$

   (c) $\lim_{x\to 3} h(x)$ if $\lim_{x\to 3} \dfrac{f(x)}{h(x)}$ does not exist

6. (9 points) In each problem below, determine whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

   (a) $\lim_{x \to -6^+} \left(\dfrac{2x+4}{x+6} \right)$

   (b) $\lim_{x \to 8} \dfrac{x^2}{(x-8)^2}$

   (c) $\lim_{x \to 7} \left(\dfrac{x}{x-7} \right)$

7. (6 points) Find and classify the discontinuities of $F(x) = \dfrac{x^2-4}{(x+3)(x-2)}$. Show work or explain your reasoning.

8. (3 points) Given that $-x^2 \leq x^2 \cos\dfrac{1}{x} \leq x^2$ for $x \neq 0$, compute $\lim\limits_{x\to 0} x^2 \cos\dfrac{1}{x}$. Explain your reasoning.

9. (2 points) Give an example of a rational function whose graph has a hole at $x = 1$ and a vertical asymptote at $x = -1$.

10. (5 points) Determine whether each statement is true (T) or false (F).

    (a) _____ If f is continuous at $x = c$, then f has a limit at $x = c$.

    (b) _____ If f has a limit at $x = c$, then f is defined at $x = c$.

    (c) _____ If f has a removable discontinuity at $x = 1$, then the limit at $x = 1$ does not exist.

    (d) _____ If $f(5) = 3$, then $\lim\limits_{x\to 5} f(x) = 3$.

    (e) _____ The limit of any basic trigonometric function can always be found by direct substitution.