Math 131 - Test 1

Name _____

September 23, 2020

Score _____

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

---

1. (6 points) Use a table of numerical values to approximate the following limit. Your table must show function values at six or more points.

$$\lim_{x \to 1} \frac{1 - 2^{x-1}}{5x - 5}$$

2. (8 points) Let $f(x) = \dfrac{x^2 - 2x - 3}{|x - 3|}$.

   (a) Compute the limit: $\lim_{x \to 3^+} f(x)$

   (b) Compute the limit: $\lim_{x \to 3^-} f(x)$

   (c) What do the results of parts (a) and (b) tell you about $\lim_{x \to 3} f(x)$?

3. (16 points) In each problem below, determine whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

   (a) $\lim_{x \to 5^+} \dfrac{x}{x-5}$

   (b) $\lim_{x \to 5^-} \dfrac{x}{x-5}$

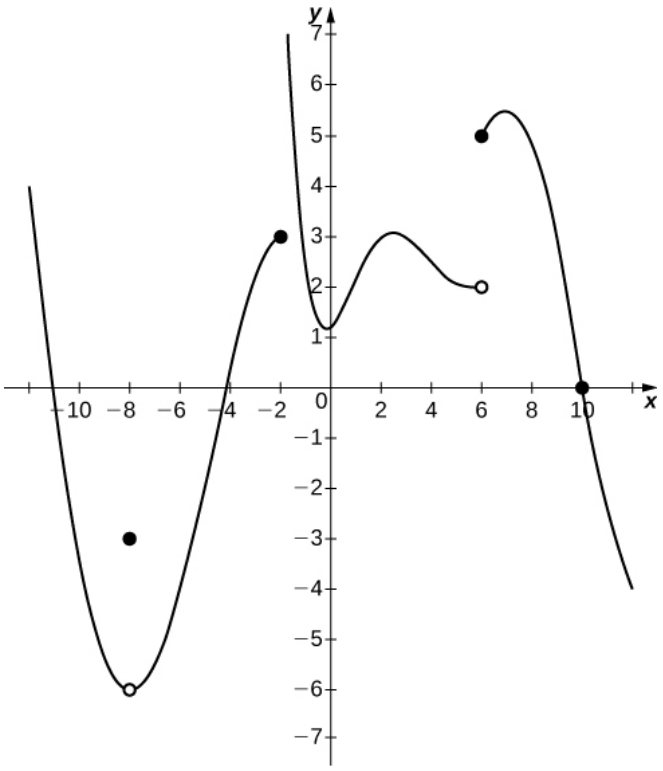
   (c) $\lim_{x \to 5} \dfrac{x}{x-5}$

   (d) $\lim_{x \to 5} \dfrac{x}{(x-5)^2}$

4. (8 points) Discuss the continuity of g. At which points is g continuous/discontinuous, and how do you know?

$$g(x) = \begin{cases} 6x + \sin(\pi x), & x \leq -1 \\ x^2 - 8x - 15, & -1 < x \leq 3 \\ x\cos(\pi x) - 9x, & x > 3 \end{cases}$$

5. (10 points) Referring to the graph of $y = f(x)$ shown below, determine each of the following or explain why it does not exist.



(a) $\lim_{x \to 10} f(x)$

(b) $f(-2)$

(c) $\lim_{x \to -8} f(x)$

(d) $\lim_{x \to -2^+} f(x)$

(e) $\lim_{x \to 6^-} f(x)$

6. (6 points) Refer to the function $y = f(x)$ whose graph is shown above. Classify the discontinuities of f.

7. (24 points) Determine each limit analytically, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE.

(a) $\lim_{x \to 0} \dfrac{\tan \pi x}{5x}$

(b) $\lim_{r \to 3^-} \left(\dfrac{r^2 - 2r - 3}{r^2 - 9} \right)$

(c) $\lim_{x \to 1} \dfrac{1 - \sqrt{x}}{1 - x}$

(d) $\lim_{t \to 0} \dfrac{(t+5)^2 - 25}{t}$

8. (6 points) Determine all vertical asymptotes of the graph of $h(x) = \dfrac{x^2 + 2x - 8}{x^2 - 4}$.
(You can use your graphing calculator for help, but you must show computational work for full credit.)

9. (6 points) Consider the following table of values for the function f.

x	-0.1	0.1	-0.01	0.01	-0.001	0.001
$f(x)$	2.8	3.2	2.98	3.02	2.998	3.002

(a) Given the table of values and no other information, is it possible that $\lim_{x \to 0} f(x)$ does not exist? Explain.

(b) Given the table of values and the fact that f is continuous everywhere, is it possible that $\lim_{x \to 0} f(x)$ does not exist? Explain.

(c) Given the table of values and the fact that f has a removable discontinuity at $x = 0$, is it possible that $\lim_{x \to 0} f(x)$ does not exist? Explain.

10. (10 points) Determine whether each statement is true (T) or false (F).

(a) _____ If f is defined at $x = c$, then $\lim_{x \to c} f(x)$ exists.

(b) _____ $\lim_{x \to 0} \sqrt{x} = 0$

(c) _____ If the graph of f has the vertical asymptote $x = 7$, then $f(7)$ is not defined.

(d) _____ If $\lim_{x \to 2} f(x) = f(2)$, then f is continuous at $x = 2$.

(e) _____ The limit of a polynomial function can always be found by direct substitution.

(f) _____ If $\lim_{x \to 3} g(x) = 7$, then $\lim_{x \to 3^+} g(x) = 7$.

(g) _____ A jump discontinuity might also be a removable discontinuity.

(h) _____ If $h(-2) = 0$, then $\lim_{x \to -2} h(x) = 0$.

(i) _____ If f and g are polynomials and $g(3) = 0$, then the graph of $\dfrac{f(x)}{g(x)}$ must have a vertical asymptote at $x = 3$.

(j) _____ The limit of a rational function can always be found by direct substitution.