Math 130 - Review 3

November 15, 2020

Name key

Score

These problems may help you review for Test 3. Your actual test will not be as long as this review packet. For other practice problems, refer to the suggested homework problems. When providing exact answers, simplify as much as possible. For each triangle described below, a is opposite α, b is opposite β, and c is opposite γ (unless otherwise indicated).

1. Find the exact solutions: $3\tan^2 x - 1 = 0$
   (Find all solutions.)

$$3\tan^2 x - 1 = 0 \Rightarrow \tan^2 x = \frac{1}{3}$$

$$\tan x = \pm\frac{1}{\sqrt{3}}$$

$$\tan x = \frac{1}{\sqrt{3}} \qquad \tan x = -\frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6} \qquad x = -\frac{\pi}{6}$$

All solutions...

$x = \frac{\pi}{6} + k\pi$, $x = -\frac{\pi}{6} + k\pi$ where k is any integer.

2. Find the exact solutions: $2\sin^2 x + 3\cos x - 3 = 0$
   (Find all solutions.)

$$2(1-\cos^2 x) + 3\cos x - 3 = 0$$

$$2 - 2\cos^2 x + 3\cos x - 3 = 0$$

$$2\cos^2 x - 3\cos x + 1 = 0$$

$$(2\cos x - 1)(\cos x - 1) = 0$$

$$\cos x = \frac{1}{2} \qquad \cos x = 1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \qquad x = 0$$

All solutions...

$x = \frac{\pi}{3} + 2k\pi$, $x = \frac{5\pi}{3} + 2k\pi$, $x = 0 + 2k\pi$

for any integer k.

3. Find the exact solutions: $2\sin 4t - \sqrt{3} = 0$
(Find all solutions.)

LET $u = 4t$ TO GET $\sin u = \frac{\sqrt{3}}{2}$.

$$\sin u = \frac{\sqrt{3}}{2} \Rightarrow u = \frac{\pi}{3}, \frac{2\pi}{3}$$

ALL SOLUTIONS IN u ...

$$u = \frac{\pi}{3} + 2k\pi, \quad u = \frac{2\pi}{3} + 2k\pi, \quad k \text{ IS ANY INTEGER.}$$

RESUBSTITUTE $(u = 4t)$ TO GET t-SOLUTIONS ...

$$\boxed{t = \frac{\pi}{12} + \frac{2k\pi}{4}, \quad t = \frac{2\pi}{12} + \frac{2k\pi}{4} \quad \text{FOR ANY INTEGER } k.}$$

4. Find the exact solutions in the interval $[0, 2\pi)$: $\sin^2 x + 2\sin x - 1 = 0$

LET $u = \sin x$

$$u^2 + 2u - 1 = 0 \Rightarrow u = \frac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2} = \frac{-2 \pm \sqrt{8}}{2}$$

$$= \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

$$\approx -2.414, \; 0.414$$

RESUBSTITUTE ...

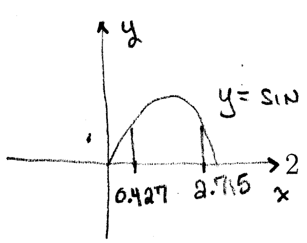
$$\underbrace{\sin x = -1 - \sqrt{2}}_{\text{NOT POSSIBLE}}$$

$$\underbrace{\sin x = -1 + \sqrt{2}}$$

$$x = \sin^{-1}(-1 + \sqrt{2}) \approx 0.427$$

AND

$$x = \pi - \sin^{-1}(-1 + \sqrt{2}) \approx 2.715$$



5. Find all solutions. Do not use a calculator.

(a) $\sin(x+\pi) - \sin x + 1 = 0$

$$\sin x \cos \pi + \cos x \sin \pi - \sin x + 1 = 0$$

$$-\sin x + 0 - \sin x + 1 = 0$$

$$-2\sin x + 1 = 0$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

ALL SOLUTIONS ...

$x = \frac{\pi}{6} + 2k\pi$, $x = \frac{5\pi}{6} + 2k\pi$, WHERE k IS ANY INTEGER.

(b) $\cos\left(x+\frac{\pi}{4}\right) - \cos\left(x-\frac{\pi}{4}\right) = 1$

$$\left(\cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4}\right) - \left(\cos x \cos\frac{\pi}{4} + \sin x \sin\frac{\pi}{4}\right) = 1$$

$$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = 1$$

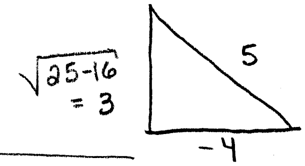
$$-\sqrt{2}\sin x = 1 \Rightarrow \sin x = -\frac{1}{\sqrt{2}}$$

$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$

ALL SOLUTIONS ...

$x = \frac{5\pi}{4} + 2k\pi$, $x = \frac{7\pi}{4} + 2k\pi$,

WHERE k IS ANY INTEGER



$\sin\beta = \frac{3}{5}$

6. Given that β is a 2nd quadrant angle with $\cos\beta = -4/5$, find the exact values of $\sin 2\beta$, $\cos 2\beta$, and $\tan 2\beta$. Do not use a calculator for this problem.

$$\sin 2\beta = 2\sin\beta\cos\beta = 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = -\frac{24}{25}$$

$$\cos 2\beta = \cos^2\beta - \sin^2\beta = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\tan 2\beta = \frac{\sin 2\beta}{\cos 2\beta} = \frac{-24/25}{7/25} = -\frac{24}{7}$$

$\frac{u}{2}$ IS A FIRST QUAD. ANGLE

7. Given that u is a 1st quadrant angle with $\cos u = 7/25$, find the exact value of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$. Do not use a calculator for this problem.

$$\sin\frac{u}{2} = +\sqrt{\frac{1-7/25}{2}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\cos\frac{u}{2} = +\sqrt{\frac{1+7/25}{2}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\tan\frac{u}{2} = \frac{\sin(u/2)}{\cos(u/2)} = \frac{3/5}{4/5} = \frac{3}{4}$$

8. Use a product-to-sum formula to rewrite $7\cos(-5x)\sin 3x$.

$$7\cos(-5x)\sin(3x) = \frac{7}{2}\left[\sin(3x-5x) + \sin(3x+5x)\right]$$

$$= \frac{7}{2}\left[\sin(-2x) + \sin(8x)\right]$$

$$= \frac{7}{2}\left[\sin 8x - \sin 2x\right]$$

9. Use a sum-to-product formula to rewrite $\cos x + \cos 4x$.

$$\cos x + \cos 4x = 2\cos\left(\frac{x+4x}{2}\right)\cos\left(\frac{x-4x}{2}\right)$$

$$= 2\cos\left(\frac{5}{2}x\right)\cos\left(-\frac{3}{2}x\right)$$

$$= 2\cos\left(\frac{5}{2}x\right)\cos\left(\frac{3}{2}x\right)$$

DOUBLE ANGLE 11. Find all solutions. Do not use a calculator.

(a) $\sin 2x - \sin x = 0$

$2 \sin x \cos x - \sin x = 0$

$\sin x \left(2\cos x - 1 \right) = 0$

$\sin x = 0$ OR $\cos x = \frac{1}{2}$

$\boxed{x = 0, \pi + 2k\pi}$ $\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3} + 2k\pi}$

SUM-TO-PRODUCT (b) $\underbrace{\sin 6x + \sin 2x}$ $= 0$

$2 \sin 4x \cdot \cos 2x = 0$

$\sin 4x = 0$ OR $\cos 2x = 0$

$4x = 0, \pi + 2k\pi$ $2x = \frac{\pi}{2}, \frac{3\pi}{2} + 2k\pi$

$\boxed{x = 0, \frac{\pi}{4} + \frac{1}{2}k\pi}$ $\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4} + k\pi}$

DOUBLE ANGLE (c) $\sin \frac{x}{2} + \cos x = 0$

$u = \frac{x}{2}$

$\sin u + \cos 2u = 0$

$\sin u + 1 - 2\sin^2 u = 0$

$2\sin^2 u - \sin u - 1 = 0$

$(2\sin u + 1)(\sin u - 1) = 0$

$\sin u = -\frac{1}{2}$ $\sin u = 1$

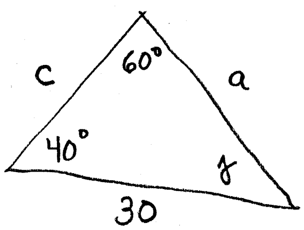
$u = \frac{7\pi}{6}, \frac{11\pi}{6} + 2k\pi$ $u = \frac{\pi}{2} + 2k\pi$

$u = x/2$

$\boxed{\begin{array}{l} x = \frac{7\pi}{3}, \frac{11\pi}{3} + 4k\pi \\ x = \pi + 4k\pi \end{array}}$

12. Solve each triangle. Round to the nearest hundredth.

(a) $\alpha = 40°$, $\beta = 60°$, $b = 30$

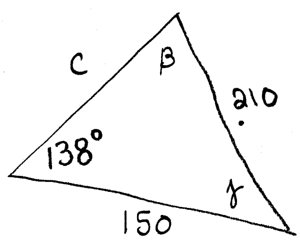


$$\frac{\sin 60°}{30} = \frac{\sin 40°}{a} \Rightarrow \boxed{a = 22.27}$$

$$\gamma = 180° - (40° + 60°) \Rightarrow \boxed{\gamma = 80°}$$

$$\frac{\sin 60°}{30} = \frac{\sin 80°}{c} \Rightarrow \boxed{c = 34.11}$$

(b) $\alpha = 138°$, $a = 210$, $b = 150$

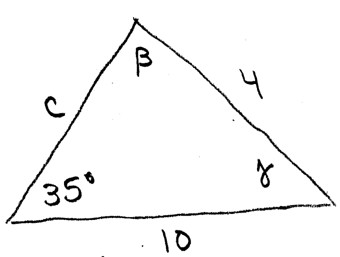


$$\frac{\sin 138°}{210} = \frac{\sin \beta}{150} \Rightarrow \beta = 28.55° \text{ or } \cancel{151.45°}$$

$$\gamma = 180° - (138° + 28.55°) \Rightarrow \gamma = 13.45°$$

$$\frac{\sin 138°}{210} = \frac{\sin 13.45°}{c} \Rightarrow c = 73.00$$

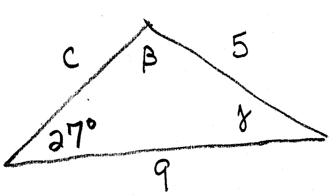
(c) $\alpha = 35°$, $a = 4$, $b = 10$



$$\frac{\sin 35°}{4} = \frac{\sin \beta}{10} \Rightarrow \underbrace{\sin \beta = 1.43}$$

No such β. ⇒ No such △.

(d) $\alpha = 27°$, $a = 5$, $c = 9$



$$\frac{\sin 27°}{5} = \frac{\sin \beta}{9} \Rightarrow \beta = 54.80° \text{ or } 125.20°$$

$\beta = 54.80°$

$$\gamma = 180° - (27° + 54.80°) \Rightarrow \gamma = 98.20°$$

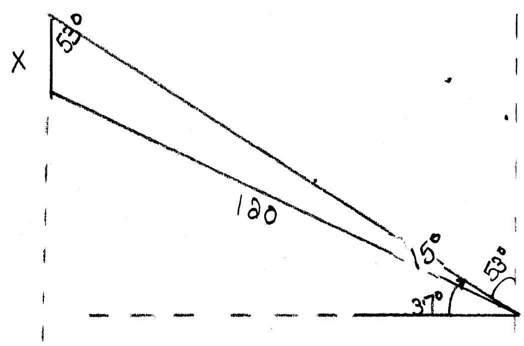
$$\frac{\sin 27°}{5} = \frac{\sin 98.20°}{c} \Rightarrow c = 10.90$$

$\beta = 125.20°$

$$\gamma = 180° - (27° + 125.20°) \Rightarrow \gamma = 27.80°$$

$$\frac{\sin 27°}{5} = \frac{\sin 27.80°}{c} \Rightarrow c = 5.14$$

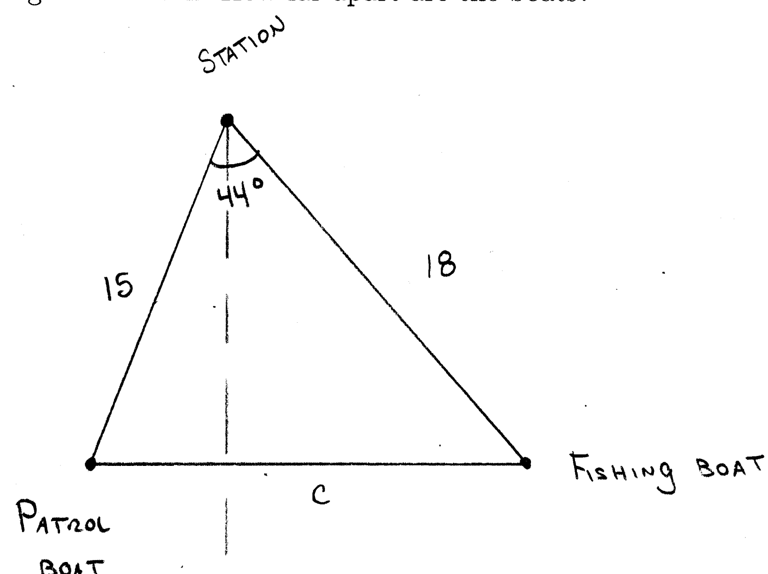
In a hilly area, a distance of 120 feet was measured down the slope of a hill from the base of a tree. From this point, the angles of elevation to the top and base of the tree are 37° and 22°, respectively. How tall is the tree?



$$\frac{\sin 53^\circ}{120} = \frac{\sin 15^\circ}{X}$$

$$\boxed{X = 38.9 \text{ FT}}$$

A fishing boat adrift at sea indicated its positions as 18 miles S 36° E from a coast guard station. A coast guard patrol boat indicated its position at 15 miles S 8° W of the coast guard station. How far apart are the boats?



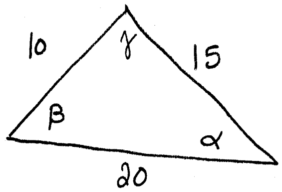
$$c^2 = 15^2 + 18^2 - 2(15)(18)\cos 44^\circ$$

$$= 160.5565\ldots$$

$$\Rightarrow \boxed{C \approx 12.7 \text{ MILES}}$$

. Solve each triangle. Round to the nearest hundredth.

(a) $a = 10,\ b = 15,\ c = 20$

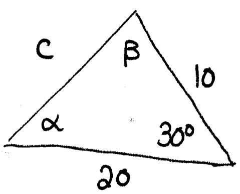


$20^2 = 10^2 + 15^2 - 2(10)(15)\cos\gamma \Rightarrow \gamma = 104.48°$

$15^2 = 10^2 + 20^2 - 2(10)(20)\cos\beta \Rightarrow \beta = 46.57°$

$10^2 = 15^2 + 20^2 - 2(15)(20)\cos\alpha \Rightarrow \alpha = 28.96°$

(b) $a = 10,\ b = 20,\ \gamma = 30°$



$c^2 = 10^2 + 20^2 - 2(10)(20)\cos 30° \Rightarrow c = 12.39$

$\dfrac{\sin 30°}{12.39} = \dfrac{\sin\beta}{20} \Rightarrow \cancel{\beta = 53.81°}$ or $\beta = 126.19°$

$\cancel{\alpha = 96.19°}$ or $\alpha = 23.81°$

No way. SHORT SIDE OPP. SMALL ∠

10. Carry out the indicated operation. Write your result in standard form.

(a) $(5 + 3i) + (2 - i)^2$

$5 + 3i + 4 - 2i - 2i + i^2 = \boxed{8 - i}$

(b) $i^5 - i(3 - 6i) = i - 3i + 6i^2 = \boxed{-6 - 2i}$

(c) $i(3 - 7i)(2 + 4i) = i(6 + 12i - 14i - 28i^2) = i(34 - 2i)$

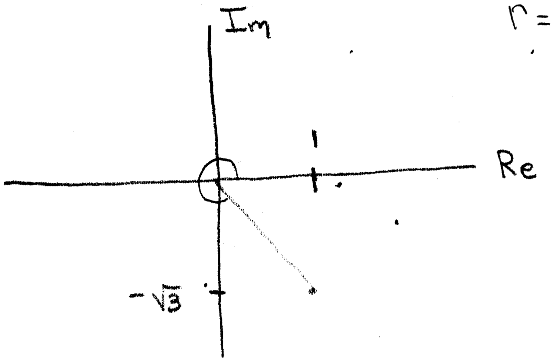
$= \boxed{2 + 34i}$

(d) $\dfrac{8 + 7i}{2i - 3}$ $\quad \dfrac{8+7i}{-3+2i} \cdot \dfrac{-3-2i}{-3-2i} = \dfrac{-24 - 16i - 21i - 14i^2}{9 - 4i^2}$

$= \dfrac{-10 - 37i}{13} = \boxed{-\dfrac{10}{13} - \dfrac{37}{13}i}$

16. Write in polar (trigonometric) form.

(a) $1-\sqrt{3}\,i$



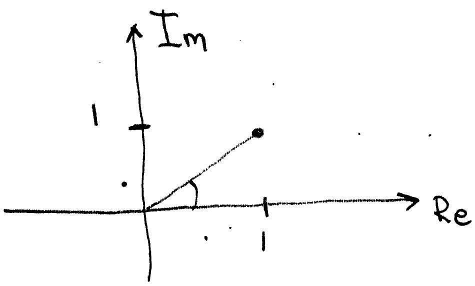
$r=\sqrt{1+3}=2$

$\tan^{-1}\left(-\frac{\sqrt{3}}{1}\right)=-60^\circ \Rightarrow \theta=300^\circ$

$2(\cos 300^\circ + i \sin 300^\circ)$

or $2 \operatorname{cis} 300^\circ$

(b) $1+i$



$r=\sqrt{1+1}=\sqrt{2}$

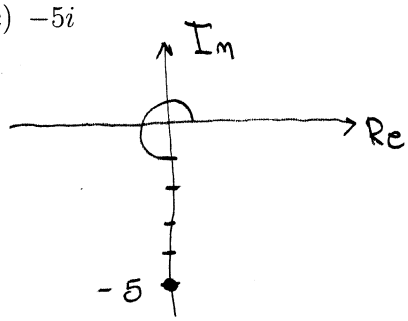
$\theta=45^\circ$

$\sqrt{2}\,(\cos 45^\circ + i \sin 45^\circ)$

or

$\sqrt{2} \operatorname{cis} 45^\circ$

(c) $-5i$



$r=5$

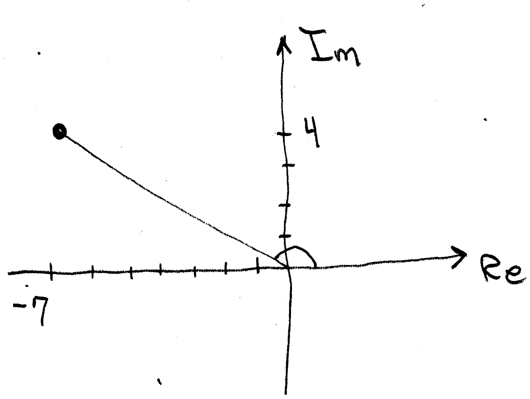
$\theta=270^\circ$

$5(\cos 270^\circ + i \sin 270^\circ)$

or

$5 \operatorname{cis} 270^\circ$

(d) $-7+4i$



$r=\sqrt{16+49}=\sqrt{65}$

$\tan^{-1}\left(\frac{4}{-7}\right)=-29.74^\circ \Rightarrow \theta=180^\circ+(-29.74^\circ)$

$=150.26^\circ$

$\sqrt{65}\,(\cos 150.26^\circ + i \sin 150.26^\circ)$

or

$\sqrt{65} \operatorname{cis} 150.26^\circ$