

## Math 129 - Review 3

November 15, 2020

Name \_\_\_\_\_

These problems may help you review for Test 3. They are coded to match the course objectives from your syllabus. Your actual test will not be as long as this review packet. Unless otherwise indicated, you should simplify all answers by reducing fractions, simplifying radicals, and/or rationalizing denominators (as you've done on your ALEKS homework). Label your axes when graphing.

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**Objective:** Evaluate difference quotients. [5]

1. Evaluate the difference quotient  $\frac{g(x+h) - g(x)}{h}$  for the function  $g(x) = 4x + 1$ .  
Completely expand and simplify your answer.

SKIP. NOT ON TEST 3.

2. Evaluate the difference quotient  $\frac{f(x+h) - f(x)}{h}$  for the function  $f(x) = x^2 + x + 2$ .  
Completely expand and simplify your answer.

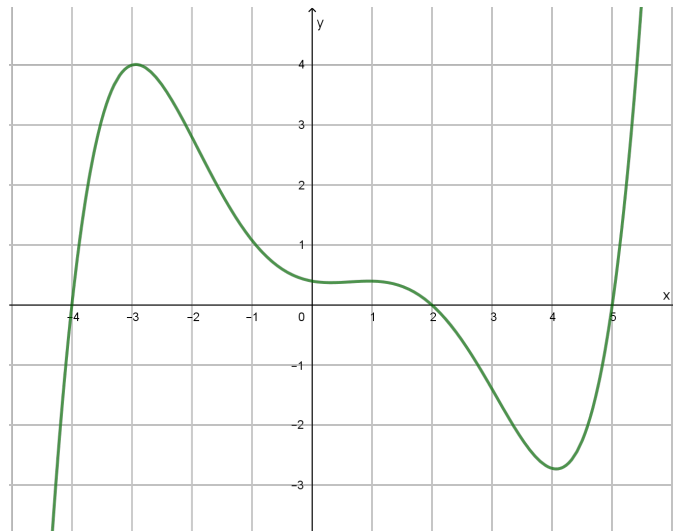
SKIP. NOT ON TEST 3.

3. Evaluate the difference quotient  $\frac{g(x+h) - g(x)}{h}$  for the function  $g(x) = 2x^2 + 6$ .  
Completely expand and simplify your answer.

SKIP. NOT ON TEST 3.

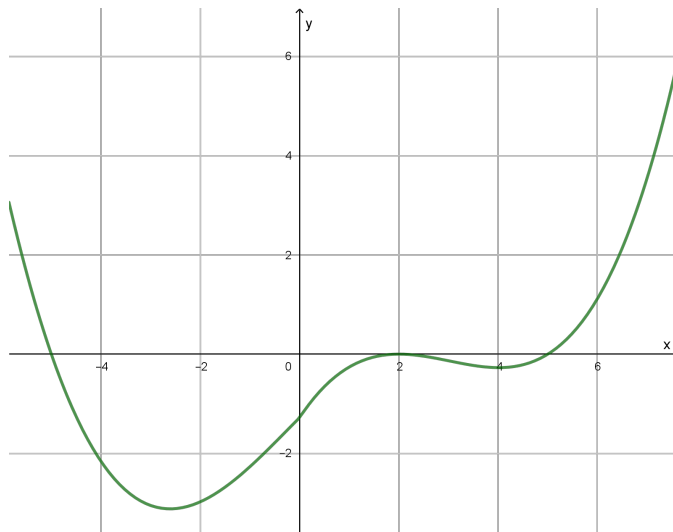
**Objective:** Given the graph of a function, determine where the function is positive, negative, or zero. [8,10]

4. The graph of the function  $f$  is shown below.



- (a) Determine the intervals on which  $f(x) > 0$ .
- (b) Determine the intervals on which  $f(x) \leq 0$ .
- (c) Determine any  $x$ -values for which  $f(x) = 0$ .

5. The graph of the function  $f$  is shown below.



- (a) Determine the intervals on which  $f(x) < 0$ .
- (b) Determine the intervals on which  $f(x) \geq 0$ .
- (c) Determine any  $x$ -values for which  $f(x) = 0$ .

**Objective:** Given the graph of a function, determine intervals on which the function is increasing, decreasing, or constant. Given the graph of a function, determine the local maxima and minima.[8,10]

6. Look back at the graph of the function in problem 4. Using the graph of  $f$ ,

(a) determine intervals on which  $f$  is increasing.

(b) determine intervals on which  $f$  is decreasing.

(c) determine all local (relative) maximum and minimum values of  $f$ .

7. Look back at the graph of the function in problem 5. Using the graph of  $f$ ,

(a) determine intervals on which  $f$  is increasing.

(b) determine intervals on which  $f$  is decreasing.

(c) determine all local (relative) maximum and minimum values of  $f$ .

**Objective:** Determine if a graph has symmetry. [8,10]

8. Given a function  $f$ , how do we determine algebraically if its graph is symmetric about the  $y$ -axis?
9. Without sketching the graph, show algebraically that the graph of  $f(x) = 3x^2 + 1$  is symmetric about the  $y$ -axis.
10. Given a function  $f$ , how do we determine algebraically if its graph is symmetric about the origin?
11. Without sketching the graph, show algebraically that the graph of  $f(x) = x^3 + 2x$  is symmetric about the origin.
12. What symmetry does the graph of  $y = 1/x$  exhibit?
13. What symmetry does the graph of  $y = 1/x^2$  exhibit?

**Objective:** Develop a familiarity with the graphs of basic functions (Toolbox Functions). [8,10]

14. For each of our basic toolbox functions (constant, linear, absolute value, squaring, cubing, square root, cube root, reciprocal, and reciprocal square functions) know each of the following:
- (a) shape of the graph,
  - (b) domain and range,
  - (c) any symmetry in the graph, and
  - (d) any vertical or horizontal asymptotes of the graph.

**Objective:** Apply the transformations (shifts, reflections, stretches, and compressions) to basic graphs to obtain more general graphs. [9]

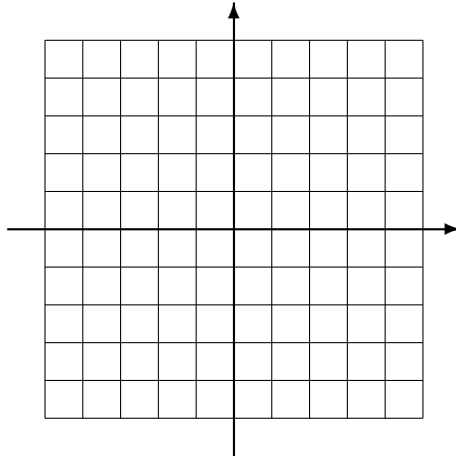
15. What are the domain and range of  $f(x) = \sqrt{x - 16} + 9$ ?

16. The graph of  $y = (x + 5)^2 - 8$  is a parabola. Where is its vertex?

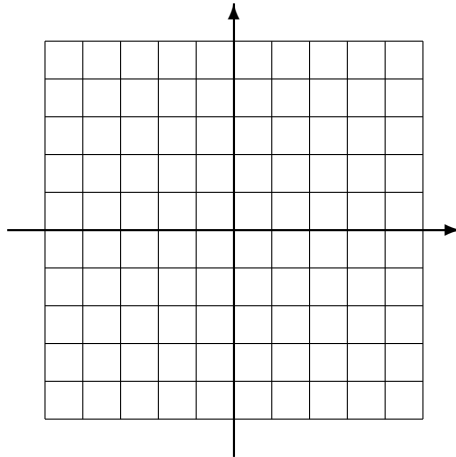
17. What are the domain and range of  $g(x) = |x + 8| + 5$ ?

18. **Carefully sketch** the graph of each function. Your graph should show details such as correct scale and position. (Label your axes.)

(a)  $g(x) = 4 - (x - 1)^2$



(b)  $f(x) = 2|x + 2| - 3$



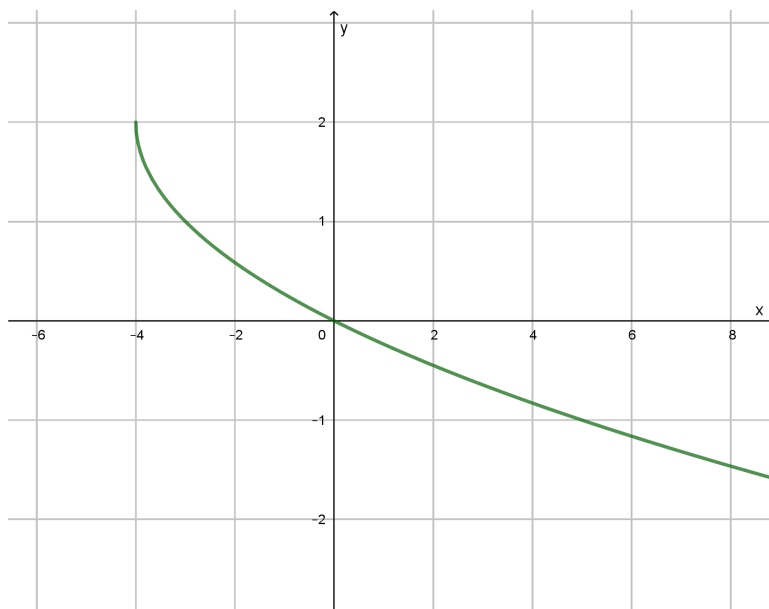
**Objective:** Determine the transformations that result in a given graph. [9]

19. Describe the sequence of transformations that transform the graph of  $y = x$  to that of  $y = (x + 4) - 5$ .

20. Describe the sequence of transformations that transform the graph of  $y = x^3$  to that of  $y = -(x - 2)^3 + 8$ .

21. How is the graph of  $y = \sqrt{x}$  related to the graph of  $y = 5\sqrt{x}$ ?

22. The graph of  $y = \sqrt{x}$  is transformed by shifts and reflections to obtain the new graph shown below. What is an equation for the new graph?



**Objective:** Develop a familiarity with basic rational functions. [10,13]

23. Describe the sequence of transformations that transform the graph of  $y = \frac{1}{x}$  to that of  $y = \frac{2}{x} + 10$ .

24. Determine the horizontal and vertical asymptotes of the graph of  $y = \frac{15}{x + 167} - 293$ .

25. Determine the horizontal and vertical asymptotes of the graph of  $y = \frac{1}{(x + 7)^2} + 8$ .



**Objective:** Define and evaluate piecewise functions. [1,5]

26. Consider the function

$$f(x) = \begin{cases} 8, & x < -1 \\ x^2, & -1 \leq x < 1 \\ \sqrt{x}, & x > 1 \end{cases}$$

(a) Evaluate  $f(5)$ .

(b) Evaluate  $f(-6)$ .

(c) Evaluate  $f(1)$

(d) What is the domain of  $f$ ?

(e) Is  $f$  a continuous function?

27. Consider the function

$$f(x) = \begin{cases} x^3 - x + 2, & x < 0 \\ x + 6, & x \geq 1 \end{cases}$$

(a) Evaluate  $f(-2)$ .

(b) Evaluate  $f(1)$ .

(c) Evaluate  $f(0)$

(d) What is the domain of  $f$ ?

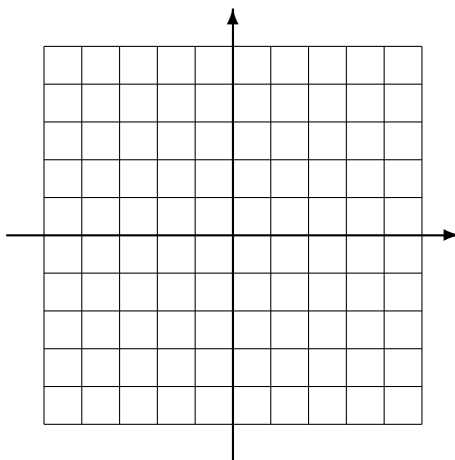
**Objective:** Sketch the graph of a piecewise-defined function. [1,5]

28. Consider the function

$$f(x) = \begin{cases} -x - 4, & x < 0 \\ 2x, & x \geq 0 \end{cases}$$

(a) What is the domain of  $f$ ?

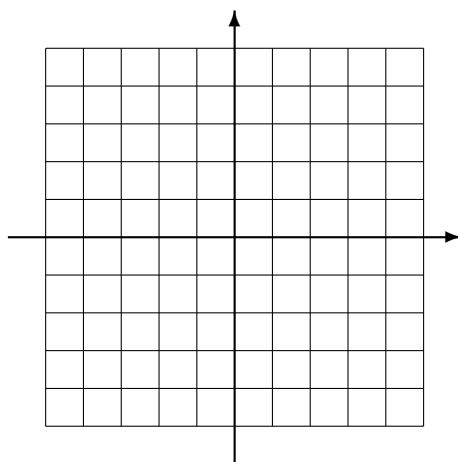
(b) Carefully sketch the graph of  $f$ . Label your axes.



(c) Is  $f$  a continuous function? If not, where is it discontinuous?

29. Sketch the graph of  $f$ . Label your axes.

$$f(x) = \begin{cases} x^2 - 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$$



**Objective:** Compute sums, differences, products, and quotients of functions. [5]

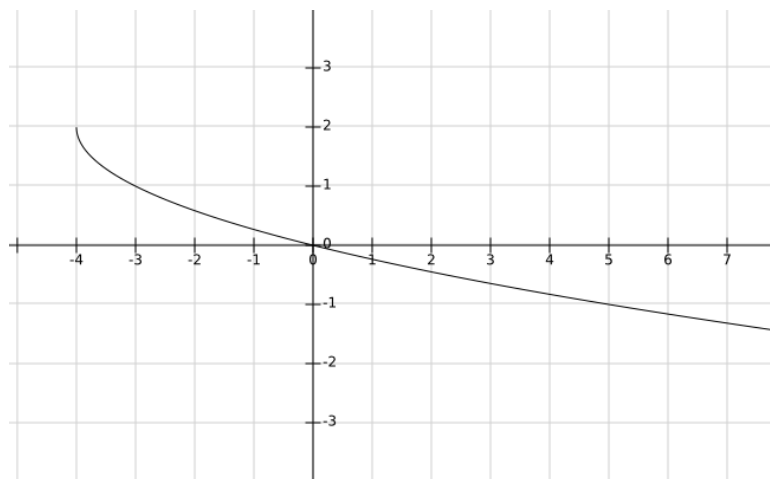
30. Let  $f(x) = 3x^2 - 8x + 1$  and  $g(x) = x^2 + 8x - \sqrt{x}$ .

(a) Find and simplify a formula for  $(f + g)(x)$ .

(b) Evaluate  $(f + g)(4)$ .

(c) Determine the domain of  $f + g$ .

31. Let  $g(x) = 3x - 5$ . Using the function  $g$  and the graph of  $y = f(x)$  shown below, compute each of the following.



(a)  $(g - f)(5)$

(b)  $(f + g)(-3)$

32. Some values of the functions  $f$  and  $g$  are given in the table below. Use the data from the table to evaluate each of the following.

$x$	0	1	2	3	4
$f(x)$	2	4	7	1	0
$g(x)$	5	0	4	8	3

(a)  $(g - f)(4)$

(b)  $(fg)(3)$

(c)  $\left(\frac{g}{f}\right)(1)$

(d)  $(f + f)(4)$

(e)  $\left(\frac{f}{g}\right)(1)$

**Objective:** Compute a composition of functions. [5]

33. Refer to the functions  $f$  and  $g$  defined in the problem above.

(a) What is the domain of  $(g \circ f)$ ?

(b) What is the range of  $(g \circ f)$ ?

34. Let  $f(x) = \sqrt{x}$  and  $g(x) = 2x + 1$ . Evaluate each of the following.

(a)  $(f \circ g)(4)$

(b)  $(g \circ f)(9)$

35. Let  $f(x) = \sqrt{2x - 4}$  and  $g(x) = 3x + 5$ .

(a) Find and simplify the formula for  $(f \circ g)(x)$ .

(b) What is the domain of  $(f \circ g)$ ?

**Objective:** Write a function as a composition of functions. [5]

36. Find two functions  $f$  and  $g$  so that  $(f \circ g)(x) = \sqrt{x^2 + x + 1}$ .

37. Find two functions  $f$  and  $g$  so that  $(f \circ g)(x) = (2x + 1)^5 + 7(2x + 1)^3$ .

**Objective:** Solve problems involving operations on functions. [5]

38. An object is cooling in such a way that its temperature in degrees Celsius after  $t$  minutes is given by  $C = 80 - 0.01t^2$ . The formula  $f = \frac{9}{5}c + 32$  is used to convert from temperatures in Celsius to Fahrenheit. Find the temperature of the object in degrees Fahrenheit at time  $t$ .