Math 112 - Test 2

March 21, 2019

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Which of these sentences are statements? Circle all that apply.

   (a) Please leave the room.

   (b) Maryam Mirzakhani is the only woman to have won the Fields Medal in mathematics.

   (c) Welcome back.

   (d) 19 is a multiple of 3.

   (e) Mathematician David Blackwell was the first African-American faculty member at UC Berkeley.

   (f) Were you late for class?

2. (5 points) Identify each as a conjunction, disjunction, conditional, or biconditional.

   (a) She finished her thesis and received her degree.

   (b) He will be the first Hispanic president if he gets elected.

   (c) I watched *Hidden Figures*, but I had already read the book.

   (d) Katherine will catch a cold if and only if she leaves the window open.

   (e) I'll ride my bike or go for a run.

3. (2 points) Write an example of a quantified statement.

4. (8 points) Write the negation of each statement in a correct sentence.

   (a) Some rectangles are not squares.

   (b) Nobody will ever run a 3-minute mile.

   (c) His last name has only two letters.

   (d) Someone in this class will get an A.

5. (6 points) Let $p =$ "Sam will get a job." and let $q =$ "Sam will buy a new car." Write each statement in symbolic form.

   (a) Sam will get a job, but he won't buy a new car.

   (b) Sam will buy a new car if he will get a job.

   (c) If Sam won't buy a new car, then he won't get a job.

6. (6 points) Refer to the statements p and q from the problem directly above. Write each statement in words.

   (a) $\sim p \wedge \sim q$

   (b) $p \longleftrightarrow p$

   (c) $\sim q \vee p$

7. (4 points) Determine the truth value of the following statement: *The current year is before 1970 if and only if there are fewer than a million people in the United States.* Show work or explain.

8. (4 points) Write the negation of the statement "Linda studies math, and Jon does not study physics." (Hint: Use DeMorgan's laws.)

9. (2 points) If q is true, what is the truth value of $\sim(\sim(\sim q))$?

10. (4 points) Without actually constructing it, determine how many rows and columns the truth table for $(p \vee q) \wedge (p \rightarrow r)$ would have. (Label which answer is which.)

11. (4 points) Write a logically equivalent statement without parentheses. (Hint: Use DeMorgan's laws.)

    (a) $\sim(p \vee q)$

    (b) $\sim(q \wedge \sim r)$

12. (2 points) True or False: The biconditional statement $p \longleftrightarrow q$ means the same as $(p \to q) \wedge (q \to p)$?

13. (5 points) Choose **any one** of the following basic operations and write its truth table: conjunction, disjunction, or conditional.

14. (6 points) Suppose p and q are false and r is true. Find the truth value of each statement. Show your work, but do not construct the entire truth table.

    (a) $p \vee (r \wedge \sim q)$.

    (b) $\sim (p \to r)$

15. (10 points) Use truth tables to show that the statement $\sim (p \to q)$ is logically equivalent to the statement $\sim q \wedge p$.

16. (8 points) Consider the following conditional statement:

If she graduates, then she starts work at NASA.

(a) State the converse.

(b) State the inverse.

(c) State the contrapositive.

(d) Which statement is logically equivalent to the original statement?

Inverse Contrapositive Converse

(e) Which statement is logically equivalent to the inverse?

Inverse Contrapositive Original

17. (3 points) There are several mistakes in the truth table shown below. Correct the mistakes.

p	q	$\sim q$	$(\sim q \vee p)$	$(\sim q \vee p) \rightarrow q$
T	T	F	F	T
T	F	T	T	F
F	T	T	F	T
F	F	T	T	T

18. (15 points) By using truth tables, determine whether each statement is a tautology, a self-contradiction, or neither.

(a) $(p \vee q) \rightarrow q$

(b) $(p \wedge q) \wedge \sim q$

(c) $p \rightarrow (p \wedge q)$