Math 112 - Test 1

February 7, 2018

Name __key__

Score _____

Show all work. Supply explanations when necessary. Partial credit will be awarded for correct work.

1. (3 points) Write the following set in set-builder notation.

$$\{\text{January, June, July}\}$$

$$= \{ x \mid x \text{ IS A MONTH THAT STARTS WITH } J \}$$

2. (3 points) Give a verbal description of the elements of the set $\{5, 10, 15, 20, \ldots\}$.

THIS IS THE SET OF NATURAL NUMBERS THAT ARE MULTIPLES OF 5.

3. (4 points) Rewrite the set S in roster notation.

$$S = \{ x \mid x \in \mathbb{N} \text{ and } 1 \le x < 7 \}$$

$$S = \{ 1, 2, 3, 4, 5, 6 \}$$

4. (5 points) Decide if each statement is true (T) or false (F).

(a) ____F____ $3.14159 \in \mathbb{N}$

(b) ____F____ $\{1, 2, 3, 4\} \subseteq \{2, 4\}$

(c) ____F____ Chicago $\in \{ x \mid x \text{ is one of the United Sates} \}$

(d) ____T____ $n(\mathbb{N}) = \infty$

(e) ____T____ $\emptyset \subseteq \{a, b, c, d, \ldots, y, z\}$

1

5. (4 points) Suppose we are thinking about the set of all patients at a doctor's office who have waited a long time to be seen. Is this set well defined? Explain why or why not.

IT IS NOT WELL DEFINED. SINCE WE DO NOT KNOW HOW MUCH TIME IS "A LONG TIME", WE CANNOT DETERMINE WHO IS IN THIS SET.

6. (2 points) Give a subset of $\{w, x, y, z\}$ that is **NOT** a proper subset.

$\{w, x, y, z\}$ IS THE ONLY SUBSET THAT IS NOT PROPER.

7. (6 points) Let A be the set of letters of the word *ILLINOIS*.

(a) Write A in roster notation.

$\{I, L, N, O, S\}$

(b) Determine $n(A)$.

$n(A) = 5$

(c) Give an example of a set that is equivalent to A, but not equal to A.

$\{1, 2, 3, 4, 5\}$

8. (5 points) For each pair of sets, determine whether they are equal, equivalent, or neither. **Circle all that apply.**

(a) $\{s, t, u, v, w\}$ $\{t, v, w, s, u\}$ (Equal) (Equivalent)   Neither

(b) $\{1, 2, 3, 4, 5, 6\}$ $\{2, 4, 6, 8, 10, 12\}$ Equal   (Equivalent)   Neither

(c) $\{five\}$ $\{f, i, v, e\}$ Equal   Equivalent   (Neither)

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9. (1 point) Which one of the following sets is NOT empty?

   (a) The set of all even numbers in $\{1, 3, 5, 7, 9, \dots\}$

   (b) $\emptyset$

   (c) The set of all natural numbers less than 1

   (d) $\{\emptyset\}$

10. (1 point) Which one of the statements below would be read "A is equivalent to B?"

   (a) $A \subseteq B$

   (b) $A \in B$

   (c) $A \cong B$

   (d) $B = \{x \mid x \in A\}$

11. (1 point) For which one of these sets is it true that $n(C) = 1$?

   (a) $C = \emptyset$

   (b) $C = \{0, 1\}$

   (c) $C = \{\emptyset\}$

   (d) $C = \{1, 10, 100, 1000, \dots\}$

12. (1 point) Let $Z = \{a, b, c\}$. Which one of these sets is NOT a proper subset of Z?

   (a) $\emptyset$

   (b) $\{a\}$

   (c) $\{a, b\}$

   (d) $\{a, b, c\}$

13. (1 point) Let $A = \{1, 3, 5, 7\}$. How many subsets does A have?

   (a) 4

   (b) 8

   (c) 15

   (d) 16

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14. (4 points) Suppose U is the set of all PSC students, and M is the subset of math students. Using words, describe the elements of M'.

M' = SET OF ALL PSC STUDENTS WHO ARE <u>NOT</u> MATH STUDENTS

15. (16 points) Let $V = \{a, e, i\}$ and $W = \{a, b, c, d, e\}$, and consider V and W as subsets of the universal set $U = \{a, b, c, d, e, f, g, h, i\}$. Determine each of the following.

(a) $n(W)$ = 5

(b) V' = $\{b, c, d, f, g, h\}$

(c) $W \cup V$ = $\{a, b, c, d, e, i\}$

(d) $W' \cap V$ = $\{i\}$ (IN V, BUT NOT IN W)

(e) $(V \cap W)'$ = $\{b, c, d, f, g, h, i\}$ (EVERYTHING EXCEPT WHAT'S COMMON TO V & W)

(f) $W \cup \emptyset$ = W = $\{a, b, c, d, e\}$

(g) $V - W$ = $\{i\}$ (SAME AS (d))

(h) $V \cap \emptyset$ = ϕ

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16. (5 points) Let $U = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$ and think about the following subsets: $A = \{14, 15, 16, 17\}$, $B = \{11, 13, 15, 17, 19\}$, $C = \{12, 14, 15, 19, 20\}$.
Determine the set $(B \cup C) \cap A'$.

$B \cup C = \{11, 13, 15, 17, 19, 12, 14, 20\}$

$A' = \{11, 12, 13, 18, 19, 20\}$

$(B \cup C) \cap A' = \{11, 13, 19, 12, 20\}$

17. (6 points) Let L be the set of letters of the word *racecar* and let $T = \{a, b\}$.

   (a) Determine the Cartesian product $T \times L$.

$L = \{a, c, e, r\}$

$T \times L = \{(a, a), (a, c), (a, e), (a, r),$
$(b, a), (b, c), (b, e), (b, r)\}$

   (b) Determine the difference $L - T$.

$L - T = \{c, e, r\}$

18. (6 points) Rewrite each of the following statements or phrases using mathematical notation.

   (a) The intersection of the sets X and Y

$X \cap Y$

   (b) The set A is equivalent to the set B.

$A \cong B$

   (c) 5 is not an element of the set Q.

$5 \notin Q$

   (d) The cardinality of the union of the two sets P and Q is 13.

$n(P \cup Q) = 13$

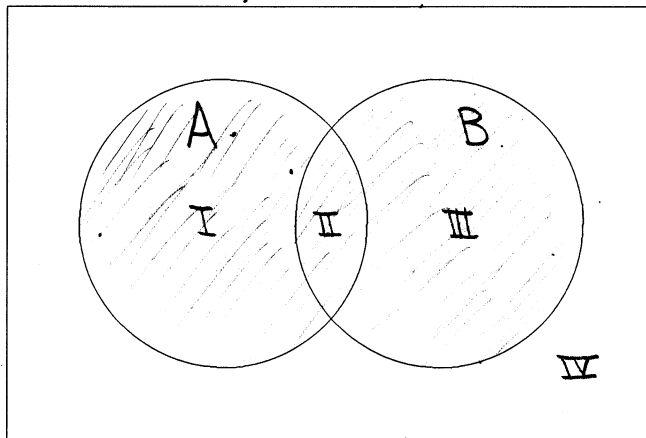   (e) The set G is a proper subset of the set H.

$G \subset H$

   (f) The set D is the empty set.

$D = \phi$

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19. (8 points) Label the sets A and B, then shade the region corresponding to each given set operation. Show work or explain your reasoning.
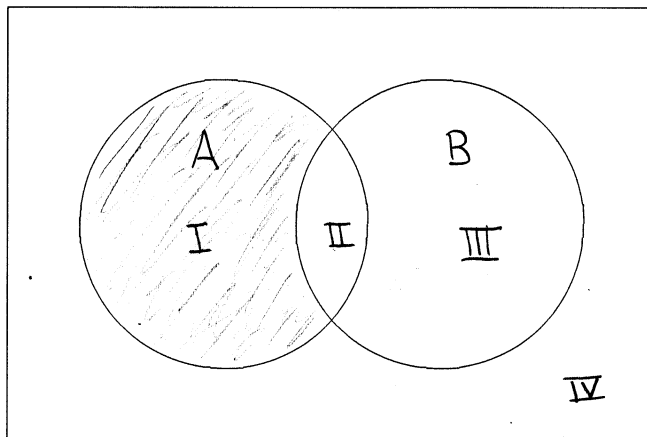
(a) $A \cup B$

$A = \{ I, II \}$

$B = \{ II, III \}$
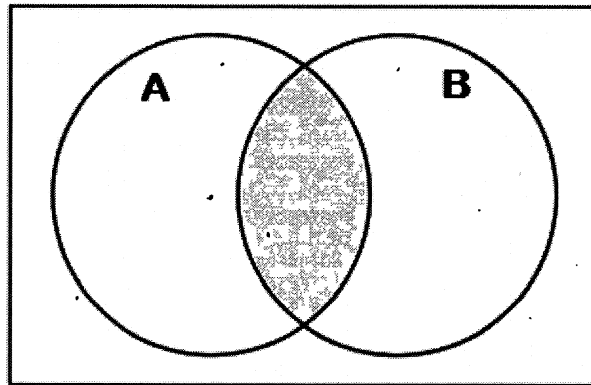
$A \cup B = \{ I, II, III \}$

(b) $A \cap B'$

$A = \{ I, II \}$

$B' = \{ I, IV \}$

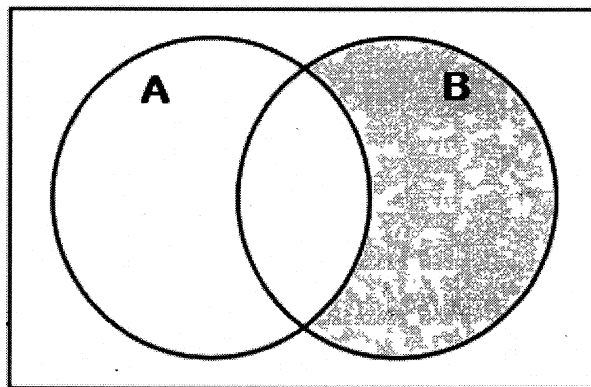$A \cap B' = \{ I \}$

20. (8 points) Use set notation to name the shaded region.
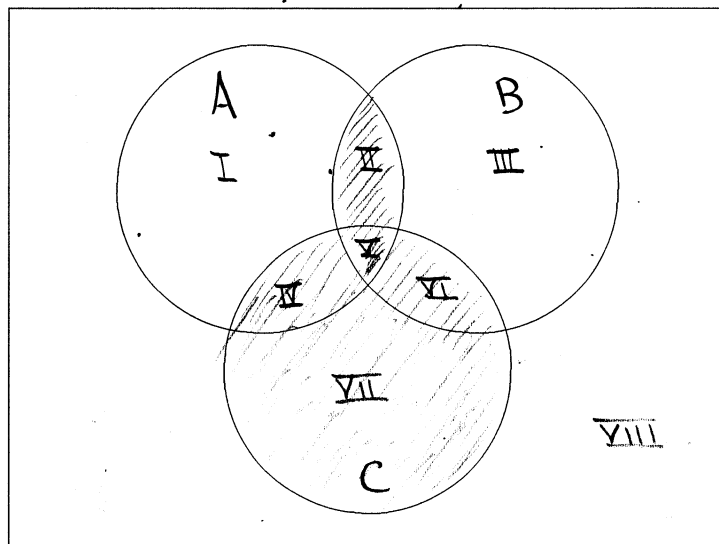

(a)

$$A \cap B$$

(b)

$$B - A \quad \text{OR} \quad B \cap A'$$

21. (10 points) Label the sets A, B, and C, then shade the region corresponding to each given set operation. Show work or explain your reasoning.

(a) $(A \cap B) \cup C$

$A \cap B = \{II, V\}$

$(A \cap B) \cup C$
$= \{II, V, IV, VI,$
$\quad VII\}$

(b) $(A - B) \cap C$

$A - B = \{I, IV\}$

$(A - B) \cap C$
$= \{IV\}$

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